SPECIFICATION OF THE MAXIMUM POSITION DURING THE DIGITIZED SIGNAL PROCESSING

Sirbu V.N.
Company Votum, Moscow, Russia

Due to the guaranteed accuracy and ideal reproducibility of results, the digital methods of signal processing are widely applied in the device manufacturing and have “De facto” become an integral part of the nondestructive testing devices. In a range of applications, for instance, in the thickness gauging, the determination of maximum position in the analog signal is of special interest meanwhile the discrete signal is accessible for processing.

The present work describes the method, enabling to determine the position of the corresponding maximum in the source analog signal by the set-up maximum in the set-up discrete signal.

Meanwhile the analog signal is discretized, the sequence of its values measured by the pre-set time step is created.

Therefore, the maximum position in the continuous analog signal will, in common, differ from the maximum position in the sequence of values of the corresponding discrete signal (fig.1) and the difference between them will be in the interval ±T/2 where T - digitization step. In a range of applications, this error can be enough great, for instance, it is equal to ±12,5nS at the discretization frequency 40MHz; in the ultrasonic thickness gauging under using of the longitudinal wave, it will make up ±0,037mm for steel what is comparable with the thickness tolerances.

Let us assume the digitized signal $Y(nT)$, $n=-N, -(N-1),\ldots,-1, 0, 1,\ldots, N-1, N$, where T — digitization period, the maximal value is reached in the point $y_0=Y(n_0T)$ by $n_0=0$ (fig.1b). There is a task to determine such a correction δ to the value n_0T that $n_0T+\delta$ sets the maximum position in the corresponding analog signal (fig.1a). Fig.1a represents the position of analog maximum indicated with the vertical line. Further, we assume that the analog signal peak is salient and symmetric relative to the selected analog maximum in some of its environment.

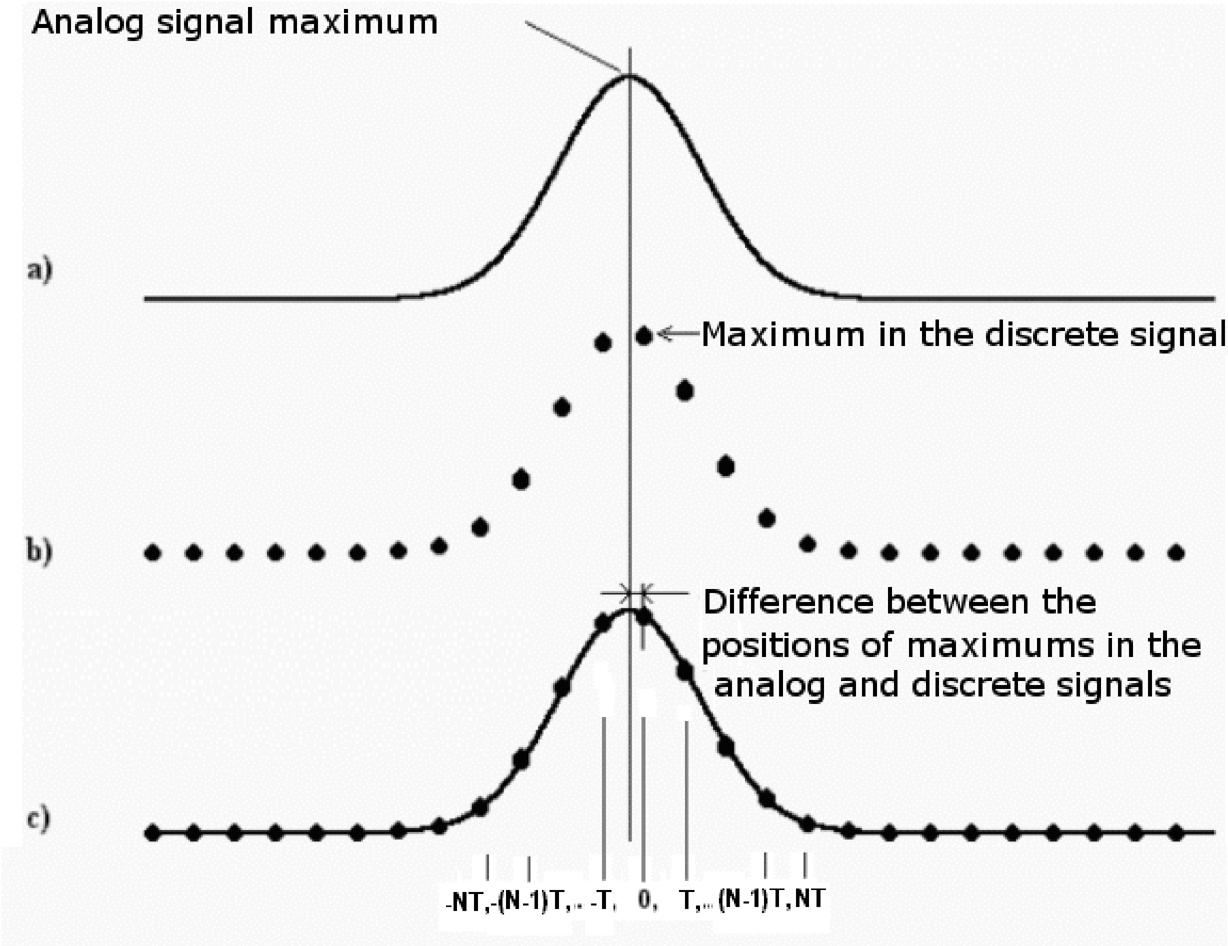

Fig. 1

Let us assume that this environment includes the following 2N+1 points of the discrete signal:

- point of the discrete maximum, having the value y_0 that corresponds to the relative time moment t=0;
- the nearest N points to the left from the discrete maximum, having the values indicated left-to-right: y_{-N}, y_{-N+1}, ..., y_{-1}, which correspond to the relative time moments t=–TN, -T(N-1), …, -T;
- the nearest N points to the right from the discrete maximum, having the values indicated left-to-right: y_1, ..., y_{N-1}, y_N, which correspond to the relative time moments t=T,…, T(N-1), TN.

Using the method of least squares, we find the coefficients *a* and *b* of the overturned parabola at^2+bt+c, giving the best approach to 2N+1 of the examined discrete signal points (1).

$$\sum_{k=-N}^{N}\left(a(kT)^2+b(kT)+c-y_k\right)^2=\min$$

$$\sum_{k=-N}^{N} \qquad (1)$$

In this case, the following expression is right for the searched correction δ

$$\delta = -b/(2a).. \tag{2}$$

To find the coefficients *a* and *b* from (1), we receive the following system of equations:

$$\begin{cases} \sum_{k=-N}^{N} 2(kT)^2 \left(a(kT)^2 + b(kT) + c - y_k\right) = 0; \\ \sum_{k=-N}^{N} 2(kT)\left(a(kT)^2 + b(kT) + c - y_k\right) = 0; \\ \sum_{k=-N}^{N} 2\left(a(kT)^2 + b(kT) + c - y_k\right) = 0. \end{cases} \tag{3}$$

The system of equations (3) has the following view

$$\begin{cases} aT^2 \sum_{k=-N}^{N} k^4 + bT \sum_{k=-N}^{N} k^3 + c \sum_{k=-N}^{N} k^2 = \sum_{k=-N}^{N} k^2 y_k; \\ aT^2 \sum_{k=-N}^{N} k^3 + bT \sum_{k=-N}^{N} k^2 + c \sum_{k=-N}^{N} k = \sum_{k=-N}^{N} k y_k; \\ aT^2 \sum_{k=-N}^{N} k^2 + bT \sum_{k=-N}^{N} k + c(2N+1) = \sum_{k=-N}^{N} y_k. \end{cases} \tag{4}$$

Taking into account that $\sum_{k=-N}^{N} k = 0$ и $\sum_{k=-N}^{N} k^3 = 0$, (4) results in

$$\begin{cases} aT^2 \sum_{k=-N}^{N} k^4 + c \sum_{k=-N}^{N} k^2 = \sum_{k=-N}^{N} k^2 y_k; \\ bT \sum_{k=-N}^{N} k^2 = \sum_{k=-N}^{N} k y_k; \\ aT^2 \sum_{k=-N}^{N} k^2 + c(2N+1) = \sum_{k=-N}^{N} y_k. \end{cases} \tag{5}$$

(5) results in the following expressions for *a* and *b:*

$$\begin{cases} b = \dfrac{1}{T} \dfrac{\sum_{k=-N}^{N} k y_k}{\sum_{k=-N}^{N} k^2}; \\ a = \dfrac{1}{T^2} \dfrac{(2N+1)\sum_{k=-N}^{N} k^2 y_k - \left(\sum_{k=-N}^{N} k^2\right)\left(\sum_{k=-N}^{N} y_k\right)}{(2N+1)\sum k^4 - \left(\sum_{k=-N}^{N} k^2\right)^2}. \end{cases} \tag{6}$$

Consequently, the expression –b/(2a) from (6) results in:

$$-\frac{b}{2a} = -\frac{T}{2} \cdot \frac{(2N+1)\sum k^4 - \left(\sum_{k=-N}^{N} k^2\right)^2}{\sum_{k=-N}^{N} k^2} \cdot \frac{\sum_{k=-N}^{N} k y_k}{(2N+1)\sum_{k=-N}^{N} k^2 y_k - \left(\sum_{k=-N}^{N} k^2\right)\left(\sum_{k=-N}^{N} y_k\right)}. \tag{7}$$

Taking into account that $\sum_{k=-N}^{N} k^2 = \frac{1}{3}N(N+1)(2N+1)$ and $\sum_{k=-N}^{N} k^4 = \frac{1}{15}N(N+1)(2N+1)(3N^2+3N-1)$, the correction's expression has the following view:

$$\delta = -\frac{b}{2a} = -\frac{T}{10}\cdot(2N+3)\cdot(2N-1)\cdot\frac{\sum_{k=-N}^{N} k\cdot y_k}{\left(3\cdot\sum_{k=-N}^{N} k^2 y_k - N(N+1)\sum_{k=-N}^{N} y_k\right)}; \qquad (8)$$

where T – digitization period, and y_{-N}, y_{-N+1}, ..., y_{-1}, y_0, y_1, ..., y_{N-1}, y_N - values of the above examined points of the discrete signal.

As a follow, (8) results in

if N=1: $$\delta = -\frac{T}{2}\cdot\frac{y_1 - y_{-1}}{y_1 - 2y_0 + y_{-1}}; \qquad (9)$$

if N=2: $$\delta = -\frac{T}{2}\cdot\frac{7}{5}\cdot\frac{2y_2 + y_1 - y_{-1} - 2y_{-2}}{2y_2 - y_1 - 2y_0 - y_{-1} + 2y_{-2}}; \qquad (10)$$

if N=3: $$\delta = -\frac{T}{2}\cdot 3\cdot\frac{3y_3 + 2y_2 + y_1 - y_{-1} - 2y_{-2} - 3y_{-3}}{5y_3 - 3y_1 - 4y_0 - 3y_{-1} + 5y_{-3}}. \qquad (11)$$

The expressions for the correction's calculation indicated in (9), (10) and (11) are simple and do not require great computational resources. Their using in the flaw detection and thickness gauging applications software of NDT devices of the company "Votum" has allowed to sufficiently increase the accuracy of measurements.