Statistical group-theoretical methods of image processing for nondestructive testing

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Introduction

This study is dedicated to development of new methods of image processing (IP) which are based on statistical estimation of invariants of group of automorphisms for image under reconstruction. For this purpose a group-theoretical statistical approach to solution of ill-posed inverse problems in IP was elaborated, which is valid for restoration of highly degraded images. Rather often, it is the only way for solution of such uncomforable problems, especially in NDT, that is why the proposed approach was first of all verified and advanced on various problems of IP in radiation testing.

Mathematical description of this or that circle of phenomena is carried out on the basis of *mathematical model* [1] which allows to draw varios theoretical conclusions which in turn can be tested then experimentally. Such extraction of consequences refers to as the solution of a *direct problems.* If the model (in particular, its functional and parametrical characteristics after comparison to experimental data) is corrected, then speak, that the *inverse problem* [1] is solved. Cycles of improvement of the model, including solution of direct and inverse problems, can be repeated. Usually, the inverse problems in NDT are sharply ill-posed. According Hadamard a problem is well-posed if its solution 1) exists, 2) it is unique, 3) it is steady. Typically, conditions 2 and 3 are broken. These circumstances stimulate the development of unified approach to IP which allows to reveal effectively characteristic features of test-object on a background of intensive noise. One of realistic ways for this is statistical revealing of group-theoretical invariants for the object within the framework of *the structural approach*.

Structure-oriented approach to image restoration

Structure in ordinary understanding is form or construction. According *American Heritage Dictionary* structure is “something made up of parts that are held or put together in a particular way”. In science and philosophy the term *structure* is treated as a set of steady connections and relations in the object which provide its wholeness and *self-identity* at various external and internal changes. Such, more strict understanding developed gradually and was finally generated in the end of XIX-th century, first in chemistry, in connection with occurrence of the “theory of chemical structure”, and then spread both in natural science, and in humanities. In the same sense the concept structure is treated in modern mathematics. It is applied to sets, in which the nature of elements is not determined. To define structure (not elements) means to set interrelations among elements, which then used as *axioms of structure*. Every change in self-identical structure is realized at the level of its elements. In mathematical models these changes are most commonly described by *group-theoretical transformations* [2]. In structure the *holistic* or “emergent”properties of object are shown. They can not be directly “derived” from properties of elements and in “structure-oriented methods” of IP [5, 7, 9, 10] they regarded as unified statistical hypotheses concerning the group of automorhisms of image. Connections and relations between elements (*structural-*

functional connections), which remains constant at transformations, refer to *structural invariants.* The prime task of classical structural approach is the revealing and classification of such invariants for a researched object and the subsequent description of this object as a structure by set of them. In principle, it can be a direct problem. But in general, especially in case this revealing involves formation and testing of hypotheses, it is an inverse problem, ill-posed or well-posed one. Its solution modifies mathematical model and enables to describe the object as new wholeness (new structure).

An primary ill-posed problem within the framework of the structural approach can be represented as deductive system with set of axioms of structure A_O – "objective statements" about test-object, which implicitly delimits the set of possible solutions of the problem In the elementary case axioms can be the initial equations of the problem (i.e. real experimental data enter into them, for example, in computerized tomography it is the measured beam-sums). They can be transformed also into an equivalent set of axioms, for example, when the problem is solved on the basis of Fourier-methods in space of frequencies, on the basis of methods of projections onto convex sets [6], methods of nonlinear backprojection [3-5, 8, 9] etc. The system of initial equations can be transformed into system of inequalities as it is done, for example, in the method of minimal projections [6] where the set of solutions is a convex set. Besides, the initial data set can be preprocessed with the purpose to cut down its internal contradictoriness, which arises due to noise in the data.

The primary ill-posed problem can not be solved. What is soluble, it is some other well-posed problem, for which the set of axioms of structure A is represented as an intersection of sets of axioms A_O and A_S, where A_S is a variable set brought by researcher in form of hypotheses i.e. $A=(A_O \cap A_S)$. The arising here subjective arbitrariness is indispensable, nevertheless it does not deprive the problem of clearness in statement. (Note, that being a product of synthetic mind, a solution of inverse problem is always a *subject-object unity*. If to use terms of Leibnitz, the solution is synthesis of *vérités de fait* and *vérités de raison* i.e.of truths of the fact (A_O) and of truths of the reason (A_S).The intersection A is not still solution of the problem, but only its ststement, the problem-solving area). .

The mathematical model within the framework of structural approach can be dererministic or statistical Too frequently, axioms A_O and A_S in deterministic models are contradictory and in case they are compatible they involve not enough serviceable information. The most flexible methods of solution of inverse problems are based on estimation of statistical compatibility of A_O and A_S and on investigation of different forms of statistical interdependence between them.

With hypotheses A_S, concerning internal organization of test-object as wholeness, in structure-oriented approach a group of transformations G_S of the object is closely related. This group generates from initial degraded image Φ (which is available from A_O) a set of images $g\Phi$. The "number" of such images is equal to order of G_S. Here g is an element of G_S. Thus, these transformations generate additional (structural) equations $g\Phi = \Phi$ which can be used for "structural regularization" of primary ill-posed problem. In general case, these equations are not exact because all the $g\Phi$ incl. Φ can be degraded by noise, incl. noise of deterministic nature as well.

The key group G_S acts as the formal tool for unification of approach to various problem-solving areas A in the "maternal" area A_O., incl. the reduced to algorithms well-posed problems, so that the group-theoretical approach admits to carry out a structural classification of problems in A_O. In other words, within the structural approach there are so much classes of inverse problems (hence, so much systems of structural equations), how much exists key groups.

The estimation of non-degraded image Ψ from the set of $g\Phi$ reminds in many respects a tomographic problem of restoration of image from a set of projections (although these two problems are not identical). In the elementary case such estimation can be received as averaging of $g\Phi$ on group (analog of backprojection) or nonlinear averaging (analog of non-linear backprojection) [8]. A particular form of non-linear backprojection is caused by a nature of noise. Generally, it is necessary to solve a joint system of initial and structural equations. For this purpose authors develop a number of procedures, both direct and iterative ones. However, it is necessary to remember, that existence of exact automorphisms of groups for the image under restoration (i.e. correspondence of structural equations to reality) is only a hypothesis (more or less succesful approximation) which should be checked.

The evaluation of approximation itself and testing of hypotheses is cumbersome in deterministic approach. On the contrary, the statistical approach allows to elaborate vigorous and flexible methods, in which the legitimacy of structural equations is considered as *statistical hypothesis*. What is more, the results of its testing (equally positive or negative) also participate in the estimation of original image Ψ and there is an opportunity to resolve fine distinctions in test-object. For this purpose in structure-oriented approach statistical techniques, both for estimation of structural invariants themselves and for estimation of their legitimacy, were developed. In particular, for evaluation of inexact automorphisms of G_S multiple *measures of similarity* μ_s (G_S, Φ) and *measures of dissimilarity* μ_d (G_S, Φ) [4, 5, 8, 9] were elaborated. They are statistics which calculated on sample set of $g\Phi$. In essence, these measures are intended for statistical evaluation of "quantity of symmetry" and "quantity of dissymmetry" in test-object. In doing so a key group G_S acts as a gauge to defines what is the symmetry itself. Most commonly, measures of similarity and dissimilarity are nonclassical statistics which are gained on the basis of non-linear backprojection) [8]. Conventional methods of mathematical statistics are applicable for their derivation as well, thus, with the use of such measures as basic tool, the apparatus of statistical support of structural invaruants is created.

For calculation of the essential characteristics of reconstructed image (incl. iimprovement of nonlinear averaging estimations) the non-normalized measures of dissimilarity are meaningful. Rather frequently, it is expedient (even necessary) to carry out these calculations not on the image as whole, but on its subsets – *regions of interes*. (ROI). A particular choice of ROI is caused by specific features of inverse problem. Besides, calculation on ROI admits to enhance accuracy of characteristics and to reduce computing expenses.

On the basis of the depicted here approach [9, 10] many specific IP techniques are developed. However, within the framework of the given publication it is necessary to concentrate attention on easily programmable methods which are based on hypotheses of "local symmetry" of structural-functional connections for evaluated image which is described by group of local symmetry L_S [9, 10]. It is a *Lie group*, in degenerated case – a finite group. In this model it is supposed, that automorphisms of group L_S hold for "background" of the image which is considered as differentiable *manifold* for which in an infinitesimal neighborhood of each point the reasons *of geometry in small* are applicable. If an operator of the local group L_S transforms one figure in this neighborhood into another, these figures are cobsidered as equivalent. The *geometry of group* (i.e. the system of statements concerning such properties of figures which are invariant with respect to transformations of group L_S) can vary (in other words it can be classical, affine, projective, conform etc.). Intuitively it is clear, that image, being the meaningful message which brings unexpectedness, cannot be reduced to quite predictable background (to "black square"), therefore in some elements of the image should be observed downturn of local symmetry i.e automorphisms of L_S are not hold here. The evaluation of a degree of lowering of local symmetry allows to form "secondary images" which in many respect are more informative than the original not degraded image. (more exactly, they bring more semantic information but less information by Shannon than this original one) If image is destructed (in particular, by noise in measuring subsystems of NDT diagnostic system) its topology is destructed as well. Nevertheless, if degree of the destruction does not exceed some “threshold of irreversibility”, then structural invariants still hold in changed degraded form, – not in infinitesimal but in some finite neighborhood of image element. In this case then image can be restored on the basis of statistical approach [9, 10], in which the presence of local symmenry of microimage in this local area is considered as *zero hypothesis* and deviation from it as *alternative hypothesis*. In doing so a measure of deviation from local symmetry (measure of dissymmetry) is constructed which used then as distribution of brightness of resulting image.

Structure-oriented filtering of projectional images of a ferro-concrete wall

Let's consider some examples of application of statistical group-theoretical IP techniques in NDT. One of the difficult and actual problems rising before “Federal Institute for Research and Testing of Materials” (“BAM”-Berlin) was the radiation testing in civil engineering, in particulat testing of building constructions with the subsequent tomographical visualization [9].

During number of years the wide program of research was carried out in BAM in this direction at technical support of Fuji Film Europe (Düsseldorf, Germany) and its European manager Dr. M.Kaling. In the problem of tomographical restoration of internal structure of a ferro-concrete wall with the limited access (a site of the old bridge) a source of radiation on a basis ^{60}Co and photoluminescent imaging plates with biomedical system “BAS2000" (Fuji Film). were used for registration of radiographic projections. The geometry of a data aqcuisition system for measurement of projections was coplanar (as in classical tomosynthesis). 7 projectional images, in the size 400×800 mm, were registered on the doubled imaging plates, the size 400×800 mm. The spatial resolution is 10 pairs lin../mm. Thickness of a wall is- 400 mm. Distance from plate up toa plane of a source is 1000 mm. The source was displaing gradually along a direct line in the source plane (with the step of 200 mm) in a direction which is perpendicular to steel rods in ferro-concrete.

As a mathematical method for tomographical restoration of the inner structure of wall nonlinear tomosynthesis [3-5, 7, 8] was quite adequate, however due to the noise, caused by scaterring radiation, essentially new methods of preprocessing of projections were required. The proposed for this aim a structure-oriented method of filtering, based on “local symmetry” [9], was successful.

There were investigated several variants of the method which yield approximately the the same results. At all variety of variants these techniques most commonly can be presented in the conventional form of algorithm of a spatial filtering with a “sliding window” which general scheme is familiar and habitual to the majority of consumers of the software for IP. In doing so a quasitomographical problem problem for estimation of the microimage inside a window arises. For continuous group L_S the "number" of generated by it microimages (or“inner views”) is continually-infinite, however in specified practical algorithms only finite subset from them is used. The set of them is used for calculation of "statistics" inside a window, most often it is the resulting brightness of the central element of image.

In building constructions the secondary elements which are responsible for certain structural-functional connections, represent extended formations and there exist certain material substructures (reinforcement in ferro-concrete) which correspond them. If such secondary elements do not give any contribution to the local projectional image inside a sliding window this image within the limits of the statistical significancr is pertinent to consider as isotropic. Otherwise there appear significant deviations from isotropy which are identified with incidence of a secondary element to the central element of the microimage. In this case the group of rotation of local image around of the central element (called also group SO (2)) serves as a group of local symmetry. It describes a background; empty space without a signal. .

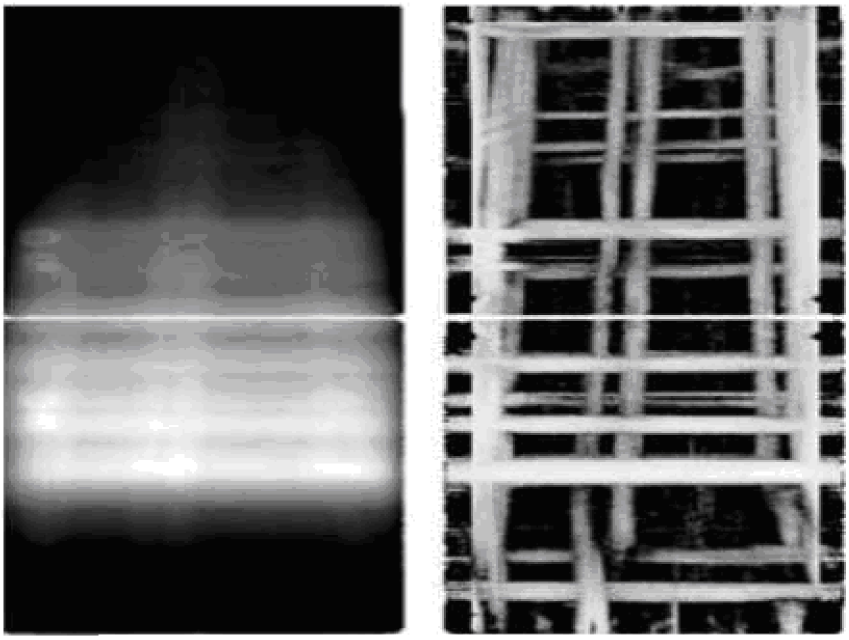
All the elements of the initial image, being the centres of local microimages and satisfying to local isotropy are regarses as equivalent. On the contrary, statistically significant anisotropy testifies the occurrence of a *semantic signal* (presenceof a secondary element, for example, reinforcement, in other cases of defect, a fragment of a crack etc.). Here, the words of P.Curie “dissymmetry creates phenomenon”, told by him in other occasion., are pertinent. Deviations from norm less often, than norm, their informational loading is greater, therefore it is natural to construct algorithm of recognition in such a manner, that symmetry describes *background* and *norm*, i. e. the most probable and trivial, that can take place. The local isotropy is considered thus, as a zero statistical group-theoretical hypothesis, and occurrence of a secondary element, as an alternative hypothesis.

Let's consider one of the algorithms of recognition of anisotropy of this class (with computing expenses close to minimal). Sample statistics for it are calculated on some subsets of the local image, namely along the direct lines which pass through the central element. . N different fixed directions which are appropriate to N groups of the data for these subsets, were considered.. For definiteness sake, we assume a sliding window square with half-width equal to M. . Let p_{ij} и r_{ij} – brightnesses of elements of initial and final images (i=1, 2,...,I, j=1,2,...,J, where I and J – dimensions of the image), n (n=1, 2, ..., N) – number of an any direction a_{1ij}, a_{2ij}, ..., a_{Nij} – average values and q_{1ij}, q_{2ij}, ..., q_{Nij} – root-mean-square in N groups of data (at calculation of these values the central element was not accounted). In order not to make the formula bulky, indexes i and j will be further omitted. We shall consider parameter n as the factor presumably influencing average values a_1, a_2, ..., a_N. According to principles of the analysis of variance the statistics – F-ratio of Fisher (i.e. intergroup variance divided by common variance inside all the groups with c N–1 and N(2M–1) degrees of freedom), which describes the resulting image, can be constructed.

$$F = \frac{N(2M-1)}{(N-1)2M} \frac{\sum_{n=1}^{N}\left(a_n - \frac{1}{N}\sum_{l=1}^{N} a_l\right)^2}{\sum_{n=1}^{N}(q_n^2 - a_n^2)},$$

More exactly, to the received formula some nonlinear transformation $r_{ij}=f(F_{ij})$ of “*look-up-table”* type can be applied in order to provide an acceptable to a human eye histogram of brightness. Thus, the final image is "depicted" by statistics of Fisher which acts in this case as a measure of dissimilarity between averages on various directions. In the given approach it is supposed, that the same statistics, on the basis of which the hypothesis is rejected or accepted, can be used as a quantitative measure of a deviation from exact symmetry (here from isotropy) as well and to serve as a characteristic of brightness of the resulting image.

In figure below results of structure-oriented filtering are submitted with the use of estimation of analysis of variance at N=4 and M=32.. On the left *(a)*— unfiltered, on the right *(b)*— filtered projections. "Black" on the filtered projection corresponds to those areas, where the zero group-theoretical hypothesis (in this case the assumption of isotropy of microimage in a local spot near to researched central element) is not rejected. On the contrary, "white" — it is the certificate of Fisher’s statistics at various levels of significance, that this zero hypothesis is not legitimate.



a *b*

Figure. Structure-oriented filtering of projection of ferro-concrete wall. Projection:a) initial ; b) filtered

At the testing of ferro-concrete constructions [9] structure-oriented filtering completely (even without tomography) solves the practical problem put by the customer, i..e. civil engineers who carry out supervision of constructions. It is pertinent to note, that the given mathematical problem is ill-posed in the sharp form, when even an operator of "blurring" of original not degraded image is practically unknown. . In fact, only one of its parameters, namely half-width M of sliding window, was used. Different authors undertook also attempts to solve this problem (i.e. a filtering of highly degraded projectional images) on the basis of well-known and new methods of image processing (the modified inverse filtration, the procedures based on fractal theory, on wavelets transformations etc.), however they have not reach positive results. The initial projectional image contains much other implicit semantic information which can be revealed with change of key group L_S. It is possible, for example, to visualize granulation in concrete if such a problem would be actual.

If to carry out analogy between the proposed here statistical group-theoretical methods and quantum physics, then it is possible to tell, that group L_S describes degenerated indistinguishable states of local microimages. In other words, "black" in a picture on the right corresponds to quite different microimages, but within the framework of the given group-theoretical model they are identical. "White" corresponds to non-degenerated states of microimages with statistically significant lowering of symmetry.

Let's to note also, that the proposed in this work synthesis of group-theoretical methods with methods of mathematical statistics expands borders of the classical structural approach and allows to solve in its framework the wide range of inverse problems.

In the case when basic theoretical results are already received, it is fruitful to simplify them up to those limits when the based on them algorithms still work, yielding the approached practical results not so strongly distinguished from present. Variants of such procedures can be found in [9].

Area of application of the developed methods: mathematical physics (in particular, for adequate description of differebt nonlinear processes and for visualisation of non-linear phenomena on the background of linear ones) computerized diagnostics and NDT (especially for solution of joint problems of NDT and materiology. The use of these methods for morphological analysis of images and for situational pattern recognition is prospective as well. .

Conclusions

1. The structure-oriented approach for solution of inverse problems in image processing is developed. Methods of spatial filtering which are based on testing of statistical hypothesis concerning local symmetry of microimage in sliding window and valid for restoration of highly degraded images, are proposed.
2. Special statistical group-theoretical algorithms of image processing which are based on situational recognition of local anisotropy of microimages with the use of properties of the group $SO(2)$ are elaborated. and found wide application in NDT.

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