SIMULATION TOOLS FOR OPTIMAL DESIGN AND INTERPRETATION OF GUIDED WAVE INSPECTIONS

A. Lhémery, K. Jezzine, V. Baronian and S. Mahaut

CEA, LIST, F-91191 Gif-sur-Yvette, France

Abstract. The paper reviews theoretical and numerical tools developed at CEA for the simulation of non-destructive testing involving guided propagation of elastic waves in the components under test. In waveguides, flaws or variations of guide shape (discontinuities) cause scattering. Incident modes reflect or are converted into new modes; their transmission through discontinuities is similarly affected. Guided waves (GW) NDT methods rely on the analysis of reflected or transmitted modes relatively to what happens in a sound guide. Simulation tools are essential to handle the inherent complexity of GW propagation and scattering; they help to optimize configurations, to interpret measurements. As GW analysis often refers to mode amplitudes, it is advantageous to exploit the modal nature of GW in the simulation itself. This simplifies the interpretation of overall or partial results (considering the various phenomena involved); this avoids developing post-processing techniques and running their time-consuming computations as required if results are calculated with no reference to modes. Two modal formulations to simulate NDT-GW measurements (pitch-catch, pulse-echo) are used to link a semi-analytic finite element code for the modal solution in arbitrary guides, models of radiation and reception by transducers and a specific finite element model for GW scattering by arbitrary discontinuities. The paper first reviews these tools. Then, examples of interest for NDT illustrate their capabilities to address complex configurations and to help interpretation.

Introduction

Elastic guided waves (GW) propagate at long range in the thickness of parts of regular shape which thickness is of the same order of magnitude as wavelengths. This property is very attractive for the nondestructive evaluation (NDE) of large structures since it limits or even avoids transducer scanning; this reduces the overall duration and cost of the examination and makes its implementation easier [1,2]; GW are also measured in NDE by Acoustic Emission (AE) of pressure vessels and can be passively and actively used in Structural Health Monitoring (SHM). Other intrinsic properties of the physical behavior of GW tend to lessen their interest. i) most GW are dispersive – their speed is frequency dependent, ii) they are multi-modal – at a given frequency, several modes coexist, their number growing with frequency; iii) modes couple when interacting with a discontinuity of the guide; iv) since their wavelength compares with structure thickness, spatial resolution is limited. All these characteristics make difficult the interpretation of results as well as the design of optimal testing configurations. Simulation tools can constitute the appropriate mean to overcome these difficulties and are expected by industrial conceivers of GW inspections. It is our objective to address these industrial needs.

In this paper, we review the modeling approach adopted for simulating NDE methods involving GW propagation and the tools used or developed at CEA to implement it. Then, advantages of the simulation approach are illustrated by examples of NDE examinations involving GW propagation.

1. Theory

1.1 Modeling approach

In general, since most guided waves are dispersive, GW testing is operated in a limited frequency bandwidth. Typically, excitation signals are in the form of a single frequency (CW) signal modulated in amplitude (Gaussian wave packets, tone bursts etc.); therefore, it is natural to model GW in the frequency domain. Typical waveforms measured are then synthesized by Fourier transform over a limited spectrum. In what follows, models are described under CW assumption where ω denotes the angular frequency.

A first property of GW propagation is that, at a given frequency, GW can be decomposed as (complex-valued) linear combination of eigenmodes peculiar to the section of the structure (perpendicular to its guiding axis denoted by z) and to its stiffness. The knowledge of the set of modes and their behavior is sufficient to depict the wave behavior of any elastodynamic quantity (particle displacement or velocity, stress) of an arbitrary field. The nth mode of this set is described at a given frequency by: i) its wavenumber β_n, real for the finite number of propagative modes, imaginary for the finite number of evanescent modes or includes an imaginary part for the infinite number of inhomogeneous modes, ii) the corresponding particle displacement vector in the invariant section of the guide $\tilde{\mathbf{u}}_\mathbf{n}(x,y)$. The CW displacement $\mathbf{u}$ associated to a wavefield writes

$$\mathbf{u}(x,y,z;t(\omega)) = \sum_n A_n \tilde{\mathbf{u}}_\mathbf{n}(x,y) e^{j(\beta_n z - \omega t)}, \quad (1)$$

where A_n denotes the nth amplitude in the decomposition. The knowledge of mode behavior and dispersion characteristics is an essential step for understanding complex phenomena arising in a guiding structure: measured or simulated signals are then interpreted in reference to modes. Typical questions concern the ability of modes to be transmitted through or reflected on a guide discontinuity, to be converted into other modes in the interaction etc. If simulated results are computed regardless of the modal nature of GW, they are very often post-processed (at a considerable computation cost) to be eventually interpreted as variations of mode amplitudes. Therefore, we made the choice to develop simulation tools fundamentally on the basis of the modal description of waves in each portion of the structure that propagates GW.

A second property of GW propagation – as applied to NDT, is that it can be described as a global phenomenon in the homogeneous portions of the structure but the way GW interact is otherwise dominated by local phenomena (e.g., transducer diffraction both in radiation and in reception, scattering by a defect, by a variation of geometrical or material properties of the structure and by any inhomogeneity of the structure). GW are especially attractive for their ability to propagate over large distances; NDT is operated by transducer(s) and aims primarily at detecting defects which are localized. Thus, the GW/NDT simulation tools must deal with different scales corresponding to various phenomena.

Moreover, industrial needs for simulation suppose that tools can be used intensively. Clearly, a single method cannot be effective for both local and global computations. We believe that various phenomena at different scales require various models. A further ingredient is necessary to give these models the possibility to work all together at the same time and in synergy. Next paragraph recalls two overall formulations [3] that constitute this central ingredient, followed by the description of models used or developed for computing modes, transducer diffraction effect on them and their scattering by inhomogeneities.

1.2 Overall Modal Formulations

Two configurations (see Fig. 1) are considered for which two formulations were derived [3]. The first (resp. second) is a pulse-echo (resp. pitch-catch) configuration. The two frequency-dependent expressions given by Eqs. (2) of the signal received $s_i(\omega)$, i=1, 2, were obtained using some mathematical properties of guided modes (bi-orthogonality) and the electro-mechanical theorem of reciprocity proposed by Auld [4].

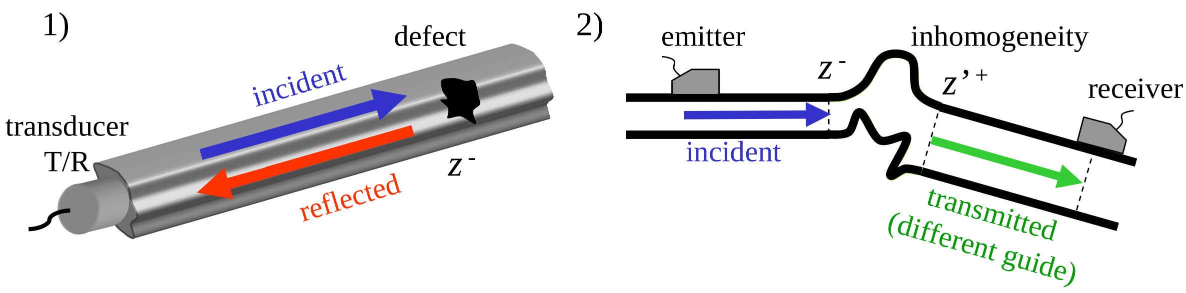

Figure 1: 1) A single transducer is used in pulse-echo configuration. 2) Two separated transducers (one emitter, one receiver) are positioned on two different waveguides in a pitch-catch configuration.

$$s_1(\omega) = \frac{-i\omega}{P} \sum_{n\in\square} \sum_{m\in\square} A_n^r A_m^e \, R_{nm} e^{i(\beta_n+\beta_m)z^-}, \tag{2a}$$

$$s_2(\omega) = \frac{-i\omega}{P} \sum_{n\in\square} \sum_{m\in\square} A_n^r A_m^e \, T_{nm} e^{i\beta_m z^-} e^{i\beta_n (L-z'^+)}, \tag{2b}$$

where P is the electrical power provided to the emitter. A_m^e and A_n^r are respectively the amplitude of mode m radiated by the transducer and the amplitude of sensitivity to mode n of the transducer in reception; they stand for transducer diffraction effects. R_{nm} (resp. T_{nm}) is the reflection (resp. transmission) coefficient for the incident mth mode and the reflected (resp. transmitted) nth mode; they stand for the scattering by an inhomogeneity of the guide(s). z^- denotes the distance between the emitter and the scattering zone; $L - z'^+$ denotes the distance between the scattering zone and the receiver in configuration 2 (along another axis z' in this case); these distances appear together multiplied by β_n, the wave number of the nth mode of a given guide, in exponential terms which are propagators of GW in the various guides involved. In practice, as soon as transducers are sufficiently distant from the scattering zone, the two discrete sums in equation (2) can be restricted to the sole propagative modes.

In these formulas, the various local phenomena (causing variation of mode amplitude) and the global propagation in homogeneous guides are mathematically separated. These formulas can admit different methods for computing the various terms of the double discrete sums. Another crucial point about them is that they make it possible to combine existing results for some of the terms with new results for other terms, opening onto vast post-processing capabilities. It is possible to use a scattering matrix with several amplitudes relative to different transducers without re-computing the whole simulation; this constitutes an economical way of optimizing testing configurations. Examples of such capabilities will be discussed in the results given in this paper.

1.3 Mode computation by the Semi-Analytical Finite Element (SAFE) method

There are many methods in the literature for computing modal solutions; some are more appropriate than others for a given application. Our aim being to offer generic tools, the semi-analytical finite element method (SAFE method, see [5] for example) appeared to be very well suited to our needs. This method involves a finite element computation in the guide section, allowing the computation of both wave vectors and modal displacements in the section as being the eigenvalues and eigenvectors (resp.) of a quadratic system of equations; this system is the discrete form of a variational problem in the guide section. Since this is a finite element computation, it allows one to deal with all sorts of characteristics of the guides (section shape, constitutive materials). As it is restricted to the section, it is computationally very efficient. The propagation is otherwise accounted for by means of analytic propagators in the guiding direction normal to the section considered, at no computational cost (same function whatever the range). In Eqs. (2), this model gives the solution for the exponential propagator terms.

1.4 Local models of transducer diffraction

There are basically two cases to distinguish: the transducer(s) can be positioned either, i) on the guiding surface; ii) on the guide section. Each case requires a specific model for computing the amplitude of the modes. Then, it is necessary to derive models adapted to the transduction that takes place; this depends on the type of transducer used (piezoelectric, EMAT, magnetostrictive). Again, this problem is addressed in the literature for various cases of industrial interest. In their present implementations, our models deal with piezo-transducers assumed to be sources of normal stresses all over their active surface. For transducers acting from the section, a specific variational formulation has been derived which is discretized on the same elements as those used for computing modes by SAFE [6]. The case of nonuniformly excited transducers – the applied stress is made variable along the active surface – has been treated allowing us to propose two methods for selecting one single mode chosen among possibly many modes [7]. For those acting from the guiding surface, a surface integration over the transducer area must be computed. In the case of an angled probe – a very common way of selecting a mode at a given working frequency by phase coincidence, it can even be computed analytically [8]. In all cases, the results are expected to be given in the form of a linear combination of modes, as in both references cited here [6, 8]. In Eqs. (2), these models give the solution for the terms A_m^e and A_n^r.

1.5 Local models of scattering by defects and by guide discontinuities

Computing the scattering by a guide inhomogeneity is a difficult task. Contrary to bulk waves typically used in NDT, guided waves have in essence a wavelength comparable with the dimensions of the guide section and with the size of the inhomogeneity that scatters GW. In the former case of bulk waves, scattering can be accurately computed by means of approximations (high frequency); in the latter case of GW, deriving suitable approximations is almost impossible.

Our aim is to write the solution of the scattering problem in the form of a matrix of complex coefficients – the reflection R_{nm} and transmission T_{nm} coefficients in Eqs. (2), assuming that modal solutions in all the guiding structures connected to the local zone of scattering are known. This matrix links an input vector constituted by the coefficients of decomposition of the incoming wave in its guiding structure, to output vectors constituted by the coefficients of decomposition of the outgoing waves in their guiding structure.

An efficient method was proposed [3] for planar cracks of arbitrary shape in an otherwise homogeneous guide, assuming that the crack surface belongs to the guide cross-section. By taking advantage of the symmetry, a variational formulation was derived, discretized on the elements of the SAFE calculation.

To deal with arbitrary flaw shapes or guide inhomogeneities (section or material variations, junction etc.), an original finite element (FE) scheme has been developed with the further goal to limit the computation zone to a minimal size, for computer efficiency. A full demonstration of the mathematical derivation is given in [9] and an extended review can be found in [10]. The computation zone being necessarily of finite size, its boundaries with all the guiding structures connected to it must be transparent for elastic waves: they must not reflect the incoming waves nor reflect outgoing waves. Thus, the main task in this development was the obtaining of artificial boundary conditions endowing transparency. Radiation conditions at infinity are brought back to the artificial boundaries by building an operator coupling the finite elements inside the FE zone to the modal solutions outside the

FE zone in the waveguides. The operator combines the displacement components with axial stresses (axes of the various guides); mathematically this is called a Dirichlet-to-Neuman operator. By doing so, an original mixed variational formulation was derived combining the displacement and a multiplier associated to the axial stresses. The scattered field is projected on modal solutions in guides through the use of bi-orthogonality relations expressed for all guides, this being done while solving the FE system. Note that it is straightforward to introduce internal sources inside the FE zone; this allows us to simulate Acoustic Emission testing for example.

2. Examples of application to NDT

One application to NDT is treated here to illustrate possible uses of the tools described. The scattering of guided waves by a junction of three identical guides is considered. The junction is considered as flaw-free in a first series of computations involving different kind of transducers acting from the guiding surface of plate #1 and operating pulse-echo configuration. Then, results are obtained in the case of the same junction which now contains a surface-breaking crack. Both kinds of results are compared. Post-processing capabilities of our simulation tools are used to study the effects of mode selection on measured waveforms.

2.1 Scattering of GW by a flaw-free junction

The junction considered (fig. 2) is that of three plates (40-mm-thick) made of steel. The radiation is operated from plate #1 and reflection and transmission from and through the junction in plates (#1-3) are studied.

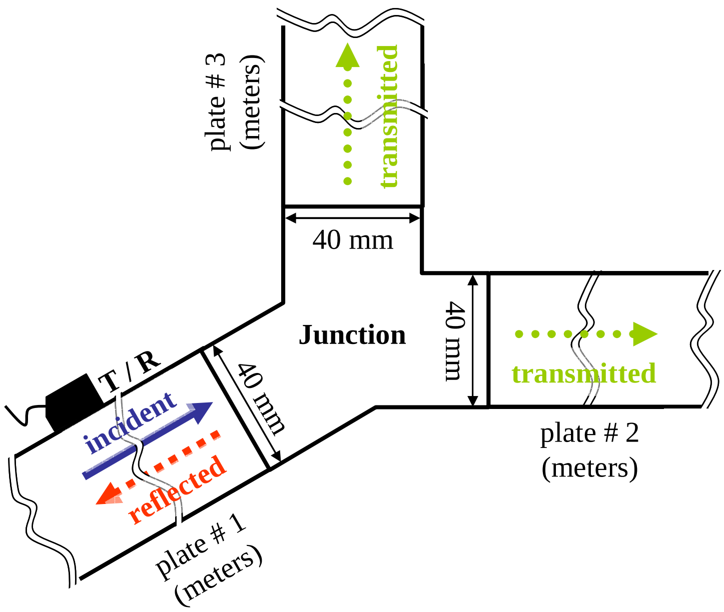

Figure 2: 1) The junction of three identical guides (plates). In plate #1, a transducer works in pulse-echo mode on the guiding surface.

The scattering matrix is computed over the bandwidth [49.5–63.5] kHz. Figure 3 shows how the various computational methods work together: in each waveguide, the modal solution is computed using the SAFE method. In the junction, the FE scheme is used. At the boundaries, as part of the FE computation, specific transparent boundary conditions are applied on which the total field is decomposed onto the modal solutions in the various uniform waveguides.

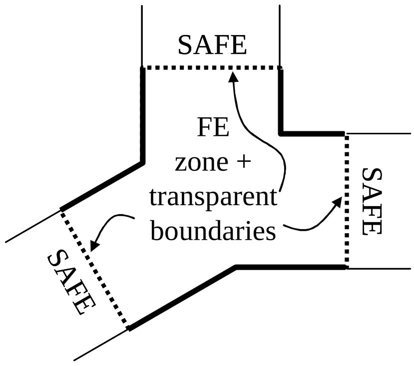

Figure 3: Detail of the computation methods used: SAFE method for the modal solutions in guides; FE method including special transparent boundary conditions in the junction.

Figure 4 shows the total field inside the junction for an incident S0 mode at the center frequency of 56.5 kHz.

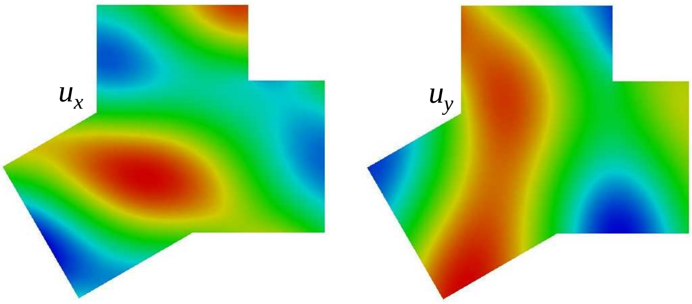

Figure 4: Total displacement field (left: u_x component, right: u_y component) in the junction considering a S0 incident mode arising from left guide, at the center frequency of 56.5 kHz.

The FE computation results in three scattering matrices (reflection, transmission) for the various possible incident modes (three propagative modes in the frequency range) scattered as propagative modes in the guides. Results are shown in Figure 5 as variations of reflection and transmission coefficients in the frequency range considered.

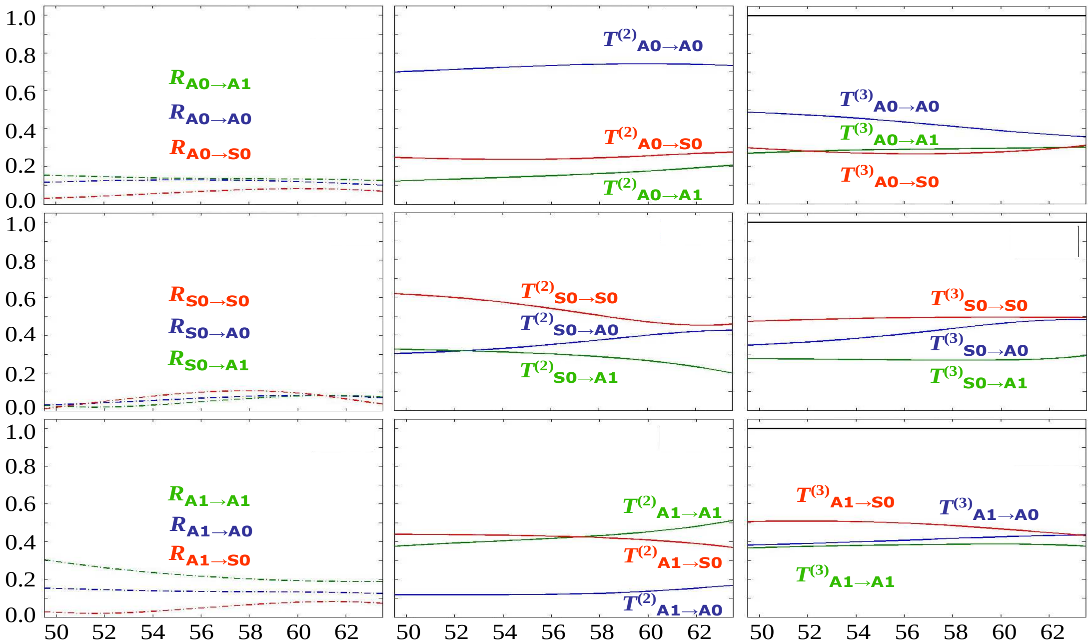

Figure 5: For A0 (top), S0 (middle) or A1 (bottom) incident mode in plate #1, scattering coefficients in plates #1-3 as functions of frequency (kHz). Left: reflection in plate #1 – Center: transmission in plate # 2 – Right: transmission in plate #3.

2.2 Scattering of GW by a junction containing a surface-breaking crack

The same junction is considered again but this time, the presence of a flaw inside the junction is simulated, as shown by Fig. 7. The flaw is a surface-breaking crack which height is half the thickness of the guiding plates linked together by the junction.

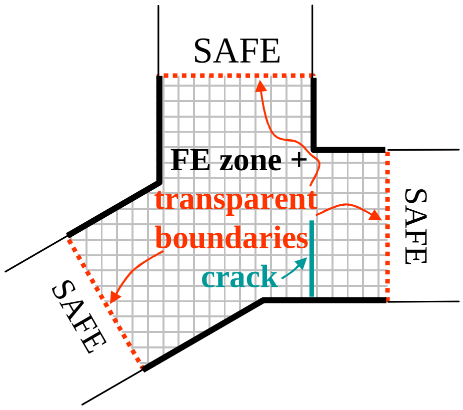

Figure 6: Same as Fig. 3, but the junction contains now a surface-breaking crack.

Figure 7 shows the total field in the junction containing the flaw, for the same conditions as those taken for the flaw-free case (shown in Fig. 4). The presence of the crack creates a strong discontinuity of the displacement which is clearly visible in the results.

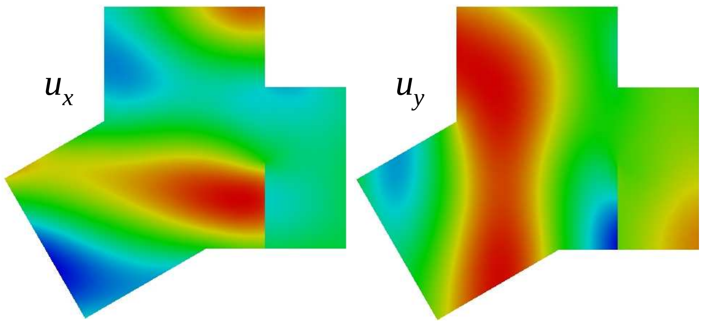

Figure 7: Same as Fig. 4, but the junction contains now a surface-breaking crack.

Similarly, the various coefficients are shown in Figure 8.

This time, reflection coefficients are of far higher amplitude, as compared to results for the flaw-free junction: the crack reflects much more energy in the guide where the incoming waves propagate (plate #1). Transmission in plate #2 decreases more than that in plate #3. Globally, the behaviour of the various coefficients as functions of the frequency becomes more complex: resonances due to the crack are likely to be at the origin of such a behaviour. Further numerical studies would be required to fully understand this behaviour as a function of the crack-length.

2.3 Synthesis of waveforms for the three transducers in pulse-echo configuration

Results for the scattering matrices are now processed by means of the overall formulas [Eqs. (2)], for various transducers. We consider three different angled-probes acting on plate #1 in pulse-echo mode. The transducer is positioned 2 meters away from the junction. Shoe angles are taken so that phase coincidence occurs between a bulk longitudinal CW in the shoe and a chosen mode at the center frequency (56.5 kHz): the angles are 70.6°, 25.2°, 19.5° for A0, S0 and A1 modes, respectively. Their active length equals 50 mm. The excitation pulse is Gaussian (10% bandwidth at -6dB).

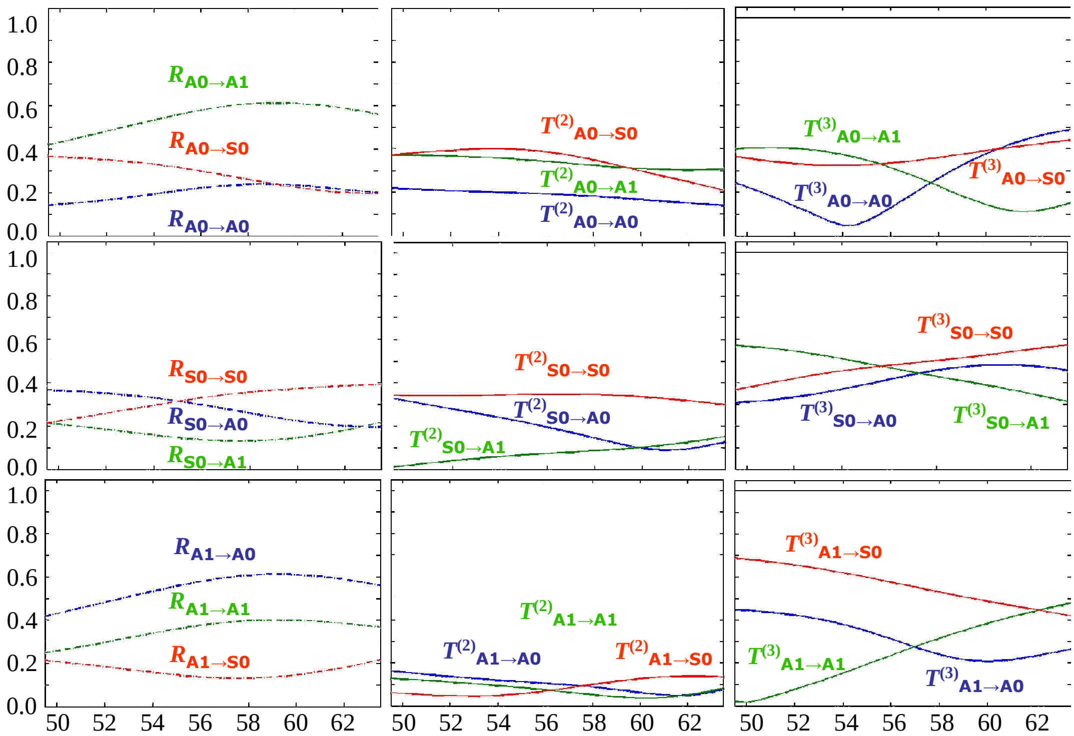

Figure 8: Same as Fig. 5, but for the junction containing a surface breaking crack.

Waveforms simulated are shown on Fig. 9. This figure compares the waveforms arising from the uncracked and the cracked junction, considering the various transducers used. The waveforms are displayed at the same amplitude scale to make the quantitative comparison easier. Note that in Eqs. (2), each term of the double sum can be seen as a signal itself. Therefore, one can easily interpret signals by decomposing them into elementary signals.

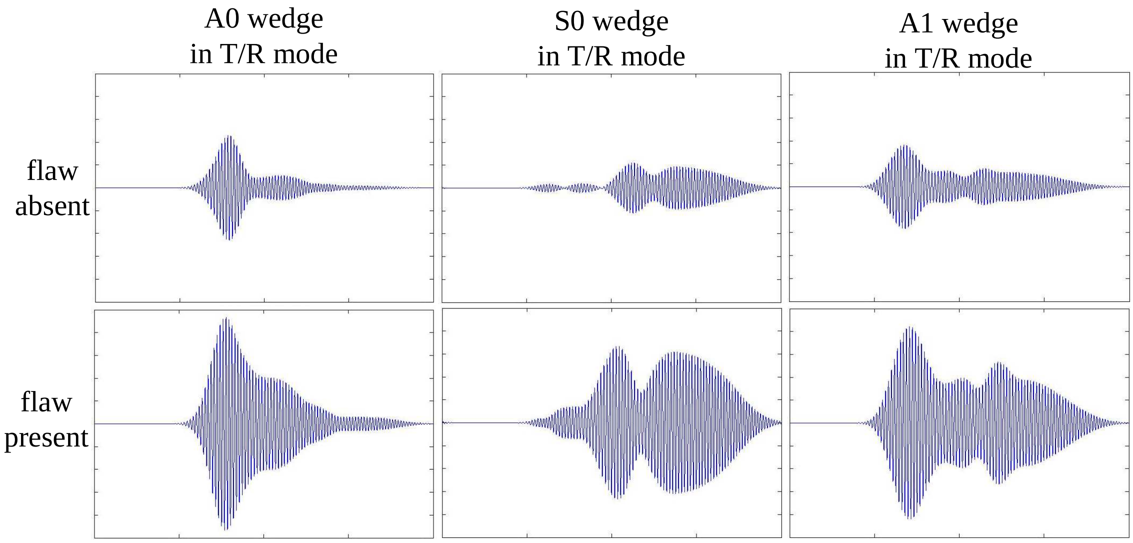

Figure 9: Waveforms received by the transducers (A0 (left column), S0 (middle column) or A1 (right column) transducers) on plate #1 used in pulse-echo. Top line: junction free of flaw. Bottom line: cracked junction. Time-scale for the various waveforms is [1.5-3.0] ms. Same amplitude scale for the various waveforms.

Even if interpreting coefficients gives a deep insight in the underlying phenomena, waveforms are what an actual measure provides; one of the interests of simulation studies is that interpreting the first kind of results (scattering coefficients) greatly helps the interpretation of waveforms. In the present results, it is obvious that reflected signals are of higher amplitude when arising from the cracked junction. But the detailed study of individual coefficients allows a better understanding of the complex phenomena that take place inside the scattering zone.

Conclusion

A modeling approach for GW / NDT simulation has been described; wavefields in guides are decomposed on modes and two formulas link local and non-local models of phenomena typical of GW measurements to deal with either pulse-echo or pitch-catch configurations. The SAFE method is used for computing long range propagation in uniform guides. GW scattering can be computed in some simple cases (crack normal to the propagation axis) by a scheme derived from SAFE. A specific FE method has been derived for computing the scattering by arbitrary inhomogeneities; it includes exact transparent artificial boundaries for reducing the size of the FE zone, thus the computation costs. Overall modal formulas offer many post-processing capabilities that help data interpretation as illustrated by some examples given herein. These tools will be implemented in future versions of CIVA software platform [11]. Experimental validation in complex cases, including applications to non-destructive testing methods such as SHM or AE are in progress.

References

[1] P. Cawley, M. J. S. Lowe, D. N. Alleyne, B. Pavlakovic and P. D. Wilcox, *Mat. Eval.* **61**, pp. 66-74, 2003.

[2] J. L. Rose, *J. Pressure Vessel Technol.* **124**, pp. 273-282, 2002.

[3] K. Jezzine and A. Lhémery, *Review of Progress in QNDE* **26**, pp. 39-46, 2007.

[4] B. A. Auld, *Wave Motion* **1**, pp. 3-10, 1979.

[5] T. Hayashi, W. J. Song and J. L. Rose, *Ultrasonics* **41**, pp. 175-183, 2003.

[6] K. Jezzine and A. Lhémery, *Review of Progress in QNDE* **25**, pp. 134-141, 2006.

[7] A. Lhémery and K. Jezzine, *Review of Progress in QNDE* **27**, pp. 123-130, 2008.

[8] J. Li and J. L. Rose, *J. Acoust. Soc. Am.* **109**, pp. 457-464, 2001.

[9] V. Baronian, A.-S. Bonnet-BenDhia and É. Lunéville, accepted for publication in *J. Comput. Appl. Math.*, 2010), on-line at http://dx.doi.org/10.1016/j.cam.2009.08.045.

[10] V. Baronian, A.-S. Bonnet-BenDhia, A. Lhémery and É. Lunéville, *Springer Proc. Phys.* **193**, p. 137-147, 2009.

[11] details on present capabilities of CIVA software platform available in www-civa.cea.fr