NON-COLLIMATED SCATTERED RADIATION TOMOGRAPHY

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For most X-ray scattering tomography, based on collimated initial and scattered radiation, the distribution of the object density (linear scattering coefficient) is reconstructed by the number of counted photons scattered in the reconstructed voxel where density distribution is received only visually and not numerically. Heterogeneous structures along the initial path and trajectory of counted scattered photons causes significant errors in imaging. Collimation of the initial and scattered radiation allows the detection of relatively few scattered photons. That is why the statistics relative to this process and the resolution are very poor. The paper presents mathematical algorithms that improve images resulting from this type of tomography and demonstrates that the imaging based on scattered collimated radiation does not provide the quality of resolution comparable to conventional transmission tomography. One approach to generate high quality scattering tomography at high resolution is the use of a large non-collimated detector. This allows counting almost all of the scattered photons leaving the object in the direction of the detector. It provide the opportunity to reconstruct the linear scattering coefficient distribution numerically.

1. Introduction

In the last decade X-ray tomography has evolved into one of the most efficient diagnostic methods available for the resolution of technical and medical problems.

There are two types of X-ray tomography:

- Transmission tomography reconstructs the distribution of the attenuation coefficient of the object.
- Back-scattered radiation tomography permits reconstruction based on the analysis of radiation scattered by the object.

The advantage of scattering tomography lies in its ability for analysis with access to only one side of the object. The present work is devoted to the development of the second type of tomography. The disadvantages are relevantly poor resolution and difficulties associated with numerical reconstruction of object internal characteristics.

Two back-scatter tomography type can be produced:

- based on collimated scattered radiation [1-8],
- based on non-collimated scattered radiation [9-14].

The present work is devoted to the development of the second type of tomography.

2. Physical bases of non-collimated scattered radiation tomography

The use of large detectors in non-collimated radiation tomography allows recording of substantial portion of scattered radiation (including multiple scattered photons). If it is possible to surround the sample with the detector, almost 100% of the photons scattered and emitted beyond the sample boundaries can be counted.

The sample is represented in the form of a collection *i,j* voxels, within which the linear scattering coefficient (density) can be taken as a constant value.

The linear scattering coefficient distribution of the sample $\mu(i,j)$ can be calculated from the arithmetical system of equations of albedo radial sums (response function)

$$\sum_{i=f(x)} \mu(i,j)\Delta L(i,j) = S(x_0, y_0, \omega_0), \qquad (1)$$

An array of albedo radial sums $S(x_0 y_0,\omega_0)$ along the initial trajectories can be obtained from the recorded scattered radiation intensity $N(x_0 y_0,\omega_0)$, measured at different angles ω_o and coordinates x_o, y_o of the initial beam radiation as:

$$S(x_0, y\ , \omega_0) = \frac{1}{k}\ln(\frac{N_0}{N_0 - \frac{k}{\alpha(x_0, y_0, \omega_0)} N_d(x_0, y_0, \omega_0)}) \qquad (2)$$

where N_0 is the initial number of photons, k is complete attenuation coefficient for Compton scatter coefficient ratio, $\alpha(x_0 y_0, \omega_0)$ is the share of recorded photons among the total amount of photons scattered in vector $(x_0 y_0, \omega_0)$.

The value of $\alpha(x_0 y_0, \omega_0)$ depends on the position and size of the detector and on the density distribution, and can be determined by simulation during the object reconstruction.

Thus, the problem is reduced to solving a linear system of equations with unknown linear scattering coefficients $\mu(i,j)$.

3. **Algorithm of reconstruction**

Let transform the matrix of linear scattering coefficiens into the vector-row. (for short note the index "C" was removed).

$$\begin{pmatrix} \mu(1,1) & \mu(1,2) & \mu(1,3) & \dots & \mu(1,N) \\ \mu(2,1) & \mu(2,2) & \mu(2,3) & \dots & \mu(2,N) \\ \mu(3,1) & \mu(3,2) & \mu(3,3) & \dots & \mu(3,N) \\ \dots & \dots & \dots & \dots & \dots \\ \mu(N,1) & \mu(N,2) & \mu(N,3) & \dots & \mu(N,N) \end{pmatrix} \Rightarrow$$

$$\Rightarrow (\mu(1,1) \quad \dots \quad \mu(1,N) \quad \mu(2,1) \quad \dots \quad \mu(2,N) \quad \dots \quad \mu(N,1) \quad \dots \quad \mu(N,N)) =$$
$$= (\mu(1) \quad \mu(2) \quad \mu(3) \quad \dots \quad \mu(P)) = \mathrm{M} \qquad (1)$$

This vector-row presents anknowing values, which must be reconstructed.

Let introduce matrixes of elementary paths in each pixel for every initial vector of X-ray beam (for various coordinates and angles) as

$$\begin{pmatrix} \Delta(1,1) & \Delta(1,2) & \Delta(1,3) & \dots & \Delta(1,N) \\ \Delta(2,1) & \Delta(2,2) & \Delta(2,3) & \dots & \Delta(2,N) \\ \Delta(3,1) & \Delta(3,2) & \Delta(3,3) & \dots & \Delta(3,N) \\ \dots & \dots & \dots & \dots & \dots \\ \Delta(N,1) & \Delta(N,2) & \Delta(N,3) & \dots & \Delta(N,N) \end{pmatrix} \Rightarrow$$

$$\Rightarrow \begin{pmatrix} \Delta(1,1) \\ \dots \\ \Delta(1,N) \\ \Delta(2,1) \\ \dots \\ \Delta(2,N) \\ \dots \\ \Delta(N,1) \\ \dots \\ \Delta(N,N) \end{pmatrix} = \begin{pmatrix} \Delta(1) \\ \Delta(2) \\ \Delta(3) \\ \\ \Delta(P) \end{pmatrix} = \Delta_k$$

(Ошибка! Текст указанного стиля в документе отсутствует.)

The most of elements of this vector are equal to zero, because the initial vector of X-ray beam crosses only small part of object pixels. In this case k-beam sum is calculated as

$$\mathrm{M}\Delta_k = S_k \ . \qquad (2)$$

During scanning the object we get the next over determined system of linear equation for each measurement

$$\Delta_1(1)\mu(1)+\Delta_1(2)\mu(2)+\Delta_1(2)\mu(2)+...\Delta_1(2)\mu(2)=S_1$$
$$\Delta_2(1)\mu(1)+\Delta_2(2)\mu(2)+\Delta_2(2)\mu(2)+...\Delta_2(2)\mu(2)=S_2$$
$$\Delta_3(1)\mu(1)+\Delta_3(2)\mu(2)+\Delta_3(2)\mu(2)+...\Delta_3(2)\mu(2)=S_3$$
....
$$\Delta_k(1)\mu(1)+\Delta_k(2)\mu(2)+\Delta_k(2)\mu(2)+...\Delta_k(2)\mu(2)=S_k$$
....
$$\Delta_q(1)\mu(1)+\Delta_q(2)\mu(2)+\Delta_q(2)\mu(2)+...\Delta_q(2)\mu(2)=S_q, \quad (3)$$

or in vector form

$$\Delta \mathrm{M} = S. \quad (4)$$

Evidently that it is necessary to have more equations that amount of pixels
q > p.
But taking into account noise-signal ration this condition must be stronger
q >> p.

The criterion functional will be

$$F = (\Delta M - S)^T (\Delta M - S) \quad (8)$$

Minimization of the functional F allows getting the best estimation of linear scattering coefficients $\tilde{M}$:

Hence the reconstruction algorithm comes to simple formula

$$\tilde{M} = WS. \quad (9)$$

4. Simulation results

Initial photons come into the object with various angles and coordinates during the scanning process. Figure 1 demonstrates the trajectories of photon pathes in backscattering tomography. If the amount of crossing paths in the reconstructed voxels in transmission tomography as a rule is constant, in backscattering tomography the deeper layer the less amount of trajectory. It decrease the object depth of reconstruction.

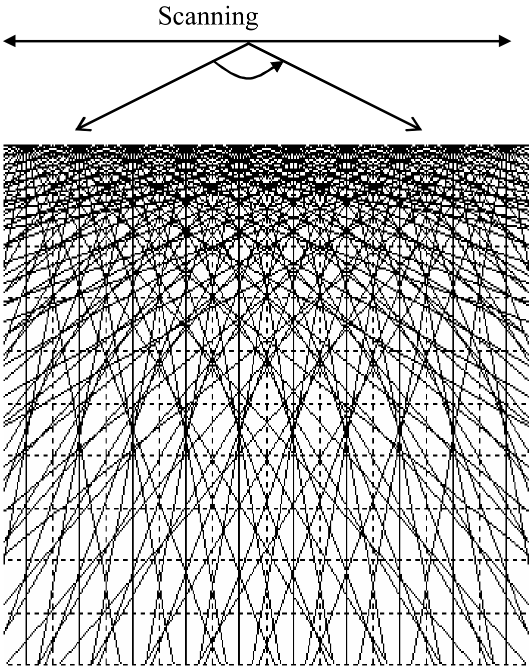

Figure 1

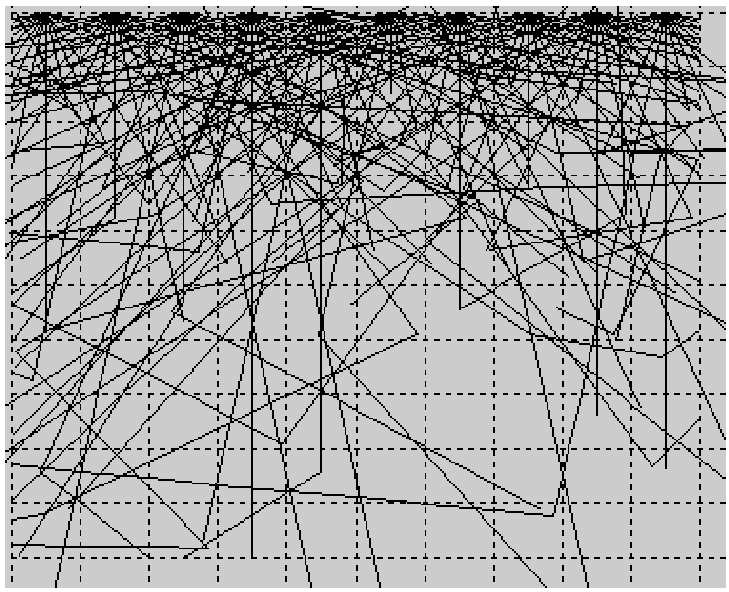

Figure 2

Another reason of depth decreasing is concluded in poor statistics of counted scattered photons. Only few part of the photons are scattered in the direction to the detector and are not absorbed at that path.

Figure 2 presents examples of simulated photon paths. It is seen that only small part of photons are counted by the detector.

The simulated scheme (Figure) consists of 4 detectors (left, right, top, bottom). The radiation is introduced from the top.

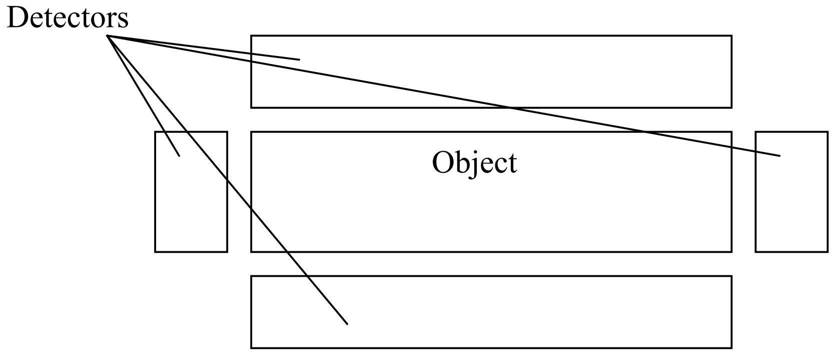

Figure 3. The scheme of detecting photon

Object (2x10 cm) consists of 20 pixels (1x1 cm) is presented in the Table 1.

Table 1. Density distribution in the object (g/cm3).

0.5	0.5	1.5	0.5	1.5	0.5	1.5	0.5	1.5	1.5
0.5	1.5	0.5	1.5	0.5	1.5	0.5	1.5	0.5	1.5

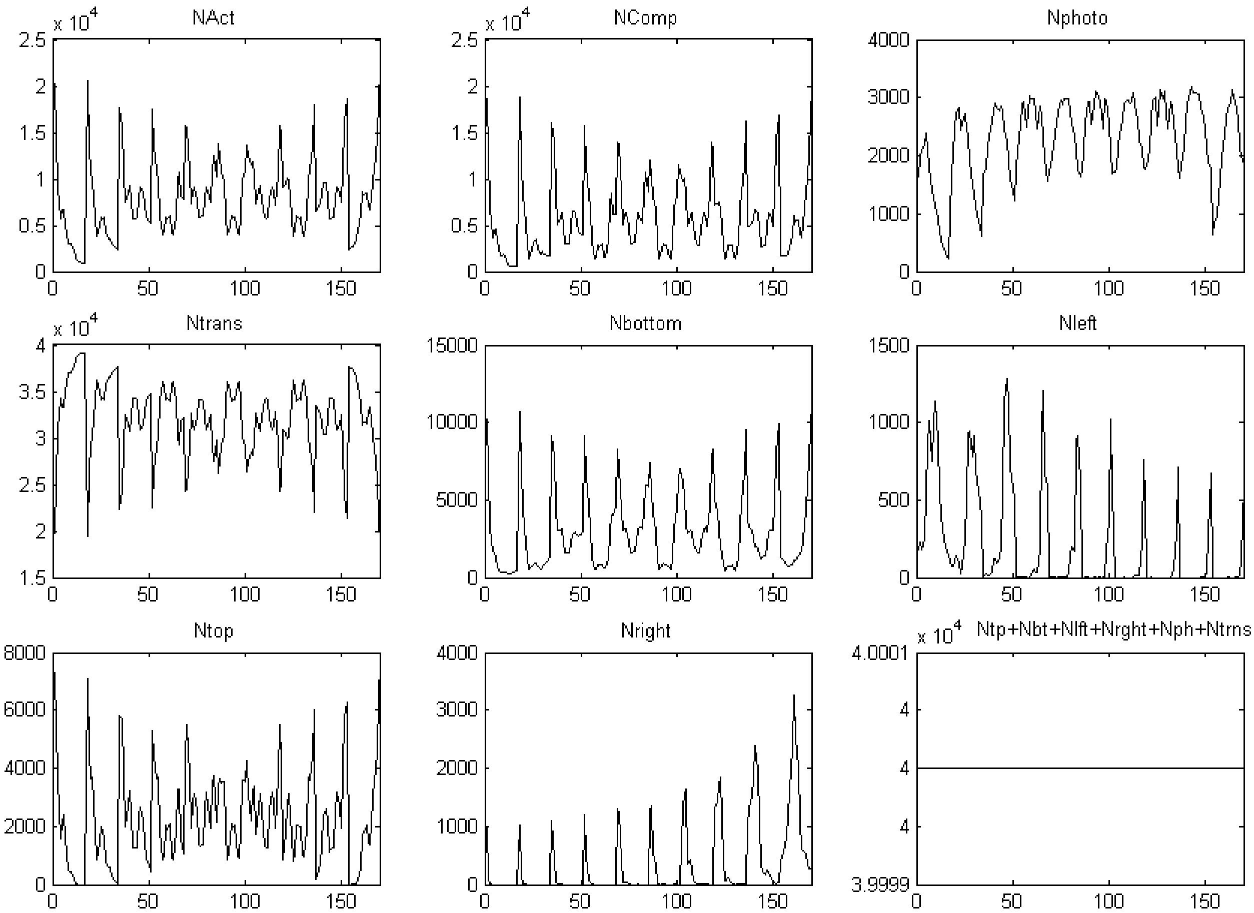

Figure 4. Amount of scatted, absorbed and left the object photons

Figure presents results of simulation: amount photons, which scatted, absorbed, and left the object.

At the figure:

Nact - amount of scatted or photo absorbed photons,

Ncomp - amount of scatted photons without photo absorption and left the object,

Nphoto - amount of photo absorbed photons,

Ntrans - amount of, which left the object without interaction (transmission photons),

Nbottom - amount of photons, which left the object at the bottom side (Nbottom does not include transmission photons),

Ntop - amount of photons, which interacted with the object left it at the top side (backscatted photons),

Nleft, Nright- amounts of photons, which left the object at the left and right sides of the object.

It is evident that

Ntop+ Nbottom+ Nleft+Nright+ Nphoto+ Ntrans=No.

X-axes at the figure corresponds scan number in the next order presented at the Table)/

Table 2. The scan consecution

Nscan	1	2	…	17	18	19	…	34	…	154	155	..	170
xo, cm	0,5				1,5				…	9,5			
φo,grad	10	20	…	170	10	20	…	170	…	10	20	…	170

Figure presents result of reconstruction on the base of scatted photon number.

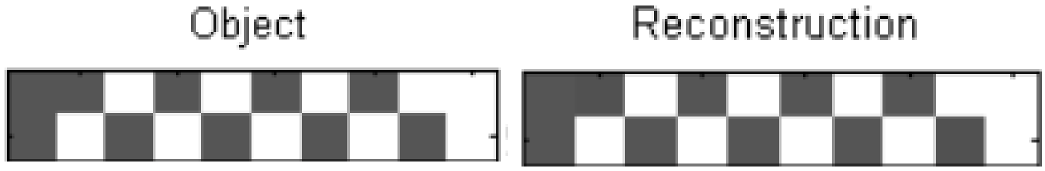

Figure 5. Result of reconstruction on the base of scatted photon number

As it is seen reconstruction of the density distribution is practically adequate to the sample.

But in reality we cannot measure the number of scatted photon. There are only counted photon left the object in our disposal.

If we knew the distribution of the density in the object we should have an opportunity of transforming amount of counted photons into the number of scatted photons. We can do it by means of Monte-Carlo simulation. But this distribution is not known. This distribution is a tag of tomography reconstruction.

But the information about the average density and the object form can be in our disposal.

By means of etalon object with constant density we can calculate the coefficient α(x0,y0,φ0), by means of which the number of counted photons Ndet(x0,y0,φ0) can be transform into the number of scatted photons Nscat(x0,y0,φ0)

$$\alpha(x_0, y_0, \varphi_0) = \frac{N_{\det}(x_0, y_0, \varphi_0)}{N_{scat}(x_0, y_0, \varphi_0)}. \qquad (10)$$

For calculation the coefficient α(x0,y0,φ0) we used the etalon homogenous object with the density 1 g/cm3 in the model Monte-Carlo.

Figure   presents amount of scatted, absorbed and left the etalon object photons.

On the base of these curves of the coefficients α(x0,y0,φ0) were calculated for every detectors (left, right, top, bottom).

Figure 7. presents examples of some coefficients curves.

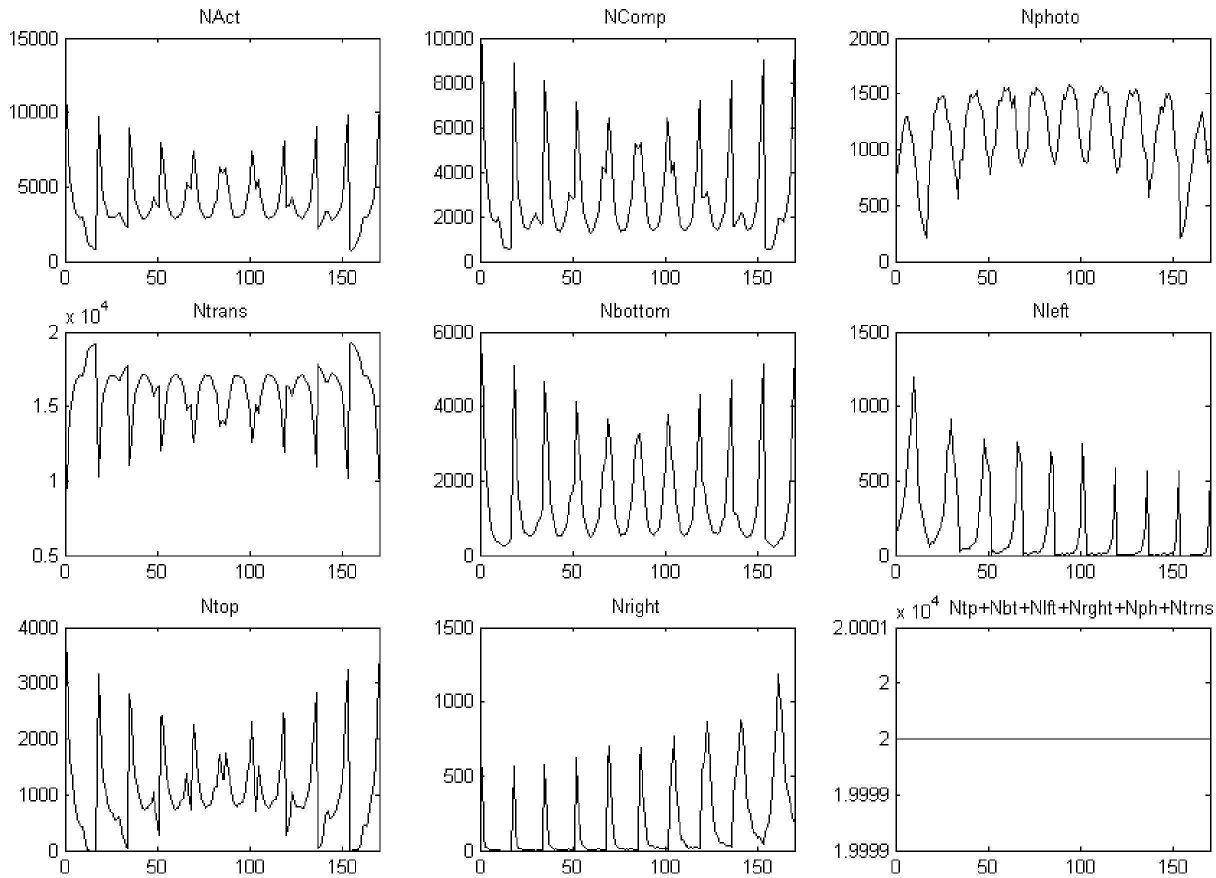

Figure 6. Amount of scatted, absorbed and left the etalon object photons

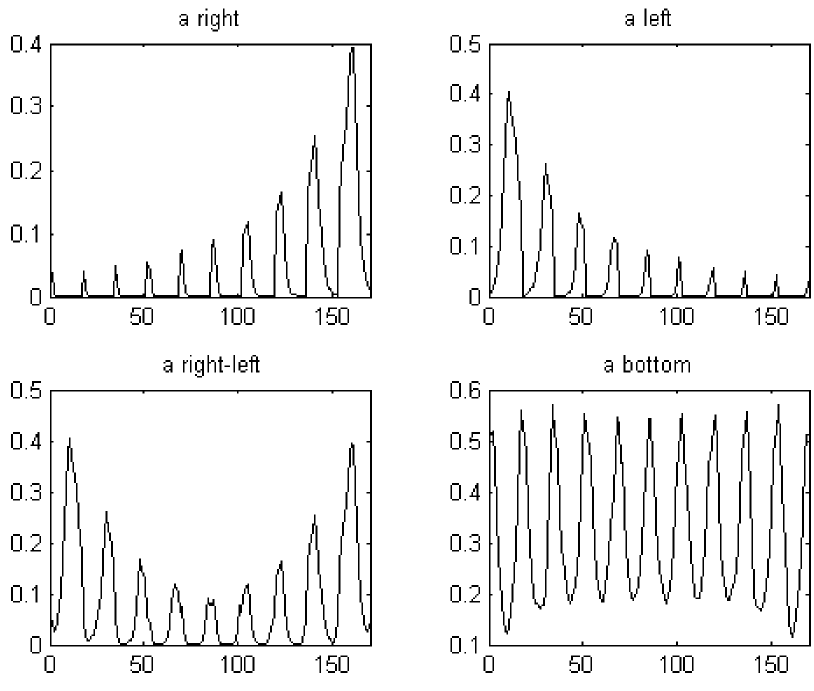

Figure 7. Coefficients α(x0,y0,φ0)

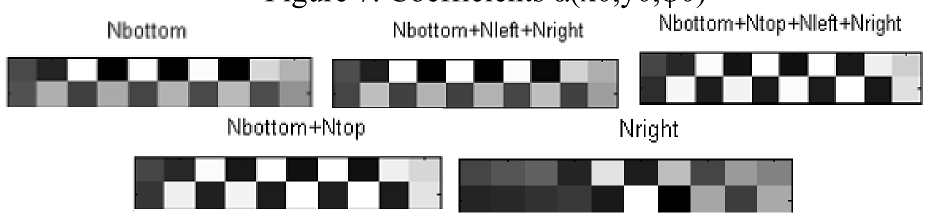

Figure 8. Reconstructions on the base of photon left the object

Figure presents reconstructions on the base of counted photons recalculated in scattered photons.

Figure 9 presents the experimental equipment which consists of collimated X-ray source (diameter of the collimator – 2 mm), the turntable (angle of rotation $\varphi_0 = 0\text{-}359^0$ with the angle step $\Delta\varphi_0 = 1^0$ and coordinate x=1-20 mm step Δx=1 mm), the detector (diameter D_D= 40 mm).

7200 scans (360x20 measurements) allowed reconstruct the sample (plastic gear with the diameter D_g = 30 mm) (Figure 1).

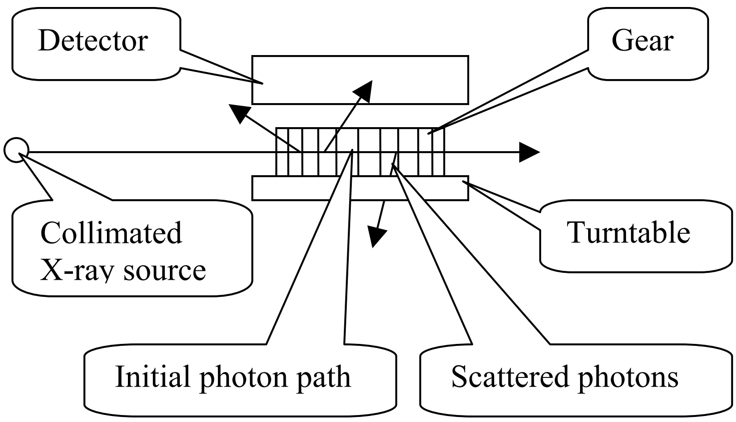

Figure 9

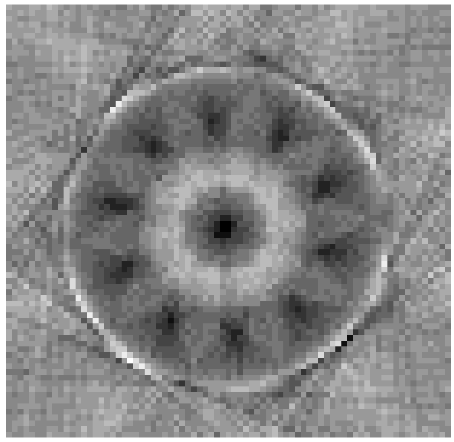

Figure 10

As it is seen the gear image is close to real sample. But there are many artifacts at the reconstruction (a ring around the gear, lines in the air etc.).

5. Conclusion

The presented theoretical and experimental results show the principle opportunity of designing the X-ray tomography on the base of non-collimated radiation.

6. References

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