UDC 534.113

DAMAGE DIAGNOSTICS IN A VERTICAL BAR ON THE ELASTIC SUSPENDER WITH CONCENTRATED MASS

M.A. Ilgamov, B.Z. Sultanov, A.N. Tazhitdinov, A.G. Khakimov
Institute of Mechanics, Ufa Branch RAS, Ufa, Russia

Using two natural longitudinal vibration frequencies we can determine both place and size of a cross incision in a vertical bar hanged on the elastic suspender and stretched by its own weight and the gravity of the load at the upper end of the bar.

Keywords: bar, vertical bar-type column, load, natural frequencies of longitudinal vibration, incision parameter, incision coordinate.

In the case of the bars of finite length for detecting defects we can use changes in the natural frequency spectrum of flexural vibrations [1] or those in natural longitudinal vibrations [2]. In [3] the solution is given for determining the variable cross-sectional area on the longitudinal coordinate by the dependence of the bar's free end movement on the perturbing force frequency. Paper [4] deals with the solution of inverse problems on longitudinal travelling waves in bars of finite length.

Consideration is given to a stress-strain state for a vertical bar hanged on the elastic suspender of rigidity c_1 and stretched by its own weight and the gravity of the load of mass M (Fig. 1). It is assumed that there is a short portion in the bar (small when compared to its total length) with a lesser cross-sectional area. This incision does not cause any bar bending and serves as a model for damage in the bar, particularly for a lateral open crack. A stress-strain state in the limits of flexibility for a thin bar is under consideration only. Considering that the crack appears as a result of the growth of a small nucleus (not necessarily in the most stressed place) we assume that the incision might be located in any place along the length of the bar. Our goal is to determine the coordinate of this incision and its size in the approximation of plane cross sections.

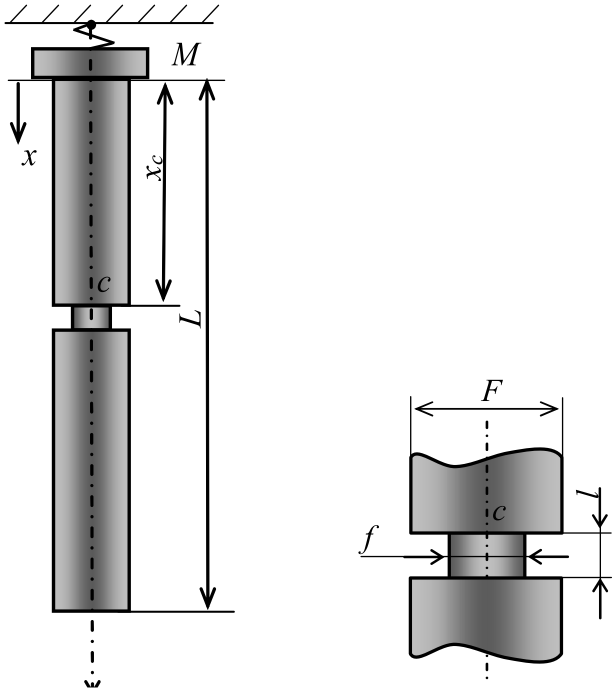

Fig. 1

Let us denote the length and cross-sectional of the bar by L, F, modulus of elasticity, density and internal friction coefficient by E, ρ, μ, length and cross-sectional area of the incision by l, f, the latter's coordinate by x_c, movement and tensile force of the bar by u, T. The relation between tension σ and deformation ε is taken as follows

$$\sigma = E\left(\varepsilon + \mu\frac{\partial\varepsilon}{\partial t}\right), \quad \varepsilon = \frac{\partial u}{\partial x}.$$

In accordance with the aforesaid we have

$$E\left(\frac{\partial^2 u}{\partial x^2} + \mu\frac{\partial^3 u}{\partial x^2 \partial t}\right) - \rho\frac{\partial^2 u}{\partial t^2} = 0, T = EF\left(\frac{\partial u}{\partial x} + \mu\frac{\partial^2 u}{\partial x \partial t}\right).$$

Measuring coordinate x from the point of fixation let us write the boundary conditions

$$T = c_1 u - M\frac{\partial^2 u}{\partial t^2} \ (x = 0), \quad T = 0 \ (x = L).$$

Within the bounds of the incision of comparatively short length l and in its close proximity there is a complex spatial stress-strain state [5]. However, for the sake of simplicity we assume uniaxial tension-compression and also take no account of inertial forces. As shown by experimental data [6], on impact at the lower end, the average value of the longitudinal damping coefficient for the hanged bar with incision is 20% greater than the same coefficient for the bar without incision. Let us examine the dynamic problem [2]

$$\frac{\partial^2 u}{\partial x^2}+\mu\frac{\partial^3 u}{\partial x^2\partial t}-\frac{\rho}{E}\cdot\frac{\partial^2 u}{\partial t^2}=0,\quad T=EF\left(\frac{\partial u}{\partial x}+\mu\frac{\partial^2 u}{\partial x\partial t}\right),$$
$$T=c_1u-M\frac{\partial^2 u}{\partial t^2}\ \left(x=0\right),\quad T=0\ \ \left(x=L\right),\tag{1}$$
$$\frac{\partial u_2}{\partial x}+\mu\frac{\partial^2 u_2}{\partial x\partial t}=\frac{\partial u_1}{\partial x}+\mu\frac{\partial^2 u_1}{\partial x\partial t},u_2-u_1=mL\frac{\partial u_1}{\partial x}-\mu\left(\frac{\partial u_2}{\partial t}-\frac{\partial u_1}{\partial t}\right),\left(x=x_c\right).$$

where

$$m=\frac{lF}{Lf}.$$

Parameter m and coordinate x_c are thus involved in the simplest model of incision. In parameter m the ratio of the cross sectional area to the length of the bar F/L is considered to be known. In case of the direct problem the ratio of the length of incision to its area is also known, the inverse problem necessitates the determination of this ratio. Variables l and f by themselves are not determined in the model [2].

The partial solution of problem (1) at $\mu=0$ has the form

$$u=\left(A\cos\alpha x+B\sin\alpha x\right)\sin\omega t\qquad\left(\alpha=\omega/a\,,a^2=E/\rho\right).$$

Four constants in this solution written for domains $0\le x\le x_c$ and $x_c\le x\le L,$ are specified by four boundary conditions (1). To make it so that A_1,A_2,B_1,B_2 will never be zero together, the following determinant should equals zero

$$\det(a_{ij})=0,\tag{2}$$

where

$$a_{11}=c_1+M\omega^2, a_{12}=-EF\alpha, a_{13}=0, a_{14}=0,$$
$$a_{21}=0, a_{22}=0, a_{23}=\sin(\alpha L), a_{24}=-\cos(\alpha L),$$
$$a_{31}=\sin(\alpha x_c), a_{32}=-\cos(\alpha x_c), a_{33}=-\sin(\alpha x_c), a_{34}=\cos(\alpha x_c),$$
$$a_{41}=-\cos(\alpha x_c)+mL\alpha\sin(\alpha x_c), a_{42}=-\sin(\alpha x_c)-mL\alpha\cos(\alpha x_c),$$
$$a_{43}=\cos(\alpha x_c), a_{44}=\sin(\alpha x_c).$$

Condition (2) gives the frequency equation

$$\begin{aligned}&mL\alpha\left(\sin^2(\alpha x_c)-\sin(\alpha x_c)\cos(\alpha x_c)\operatorname{tg}(\alpha L)\right)+\\ &+mL\alpha\left(\operatorname{tg}(\alpha L)-\sin(\alpha x_c)\cos(\alpha x_c)-\operatorname{tg}(\alpha L)\sin^2(\alpha x_c)\right)(c+D)+\\ &+\operatorname{tg}(\alpha L)-c-D=0,\ c=\frac{c_1}{EF\alpha},\ D=\frac{M\omega^2}{EF\alpha}.\end{aligned} \tag{3}$$

When coefficient $c\to\infty$, the frequency equation takes the form

$$mL\alpha\left(\operatorname{tg}(\alpha L)-\sin(\alpha x_c)\cos(\alpha x_c)-\operatorname{tg}(\alpha L)\sin^2(\alpha x_c)\right)-1=0. \tag{4}$$

In this case load mass M has no effect on natural vibration frequencies of the bar. The term involving m in (4) gives a change in these frequencies.

At the known coordinate of the incision parameter m is determined by the formula

$$m=\frac{c+D-\operatorname{tg}(\alpha L)}{\alpha L\left(d_1+d_2(c+D)\right)},$$
$$d_1=\sin^2(\alpha x_c)-\sin(\alpha x_c)\cos(\alpha x_c)\operatorname{tg}(\alpha L),$$
$$d_2=\operatorname{tg}(\alpha L)\cos^2(\alpha x_c)-\sin(\alpha x_c)\cos(\alpha x_c).$$

At $c\to\infty$

$$m=\frac{1}{\alpha L\left(\operatorname{tg}(\alpha L)\cos^2(\alpha x_c)-\sin(\alpha x_c)\cos(\alpha x_c)\right)}.$$

The mass of the load hanged on the bar can be determined by the formula

$$M=\frac{EF\alpha\left[\operatorname{tg}(\alpha L)-c+mL\alpha(d_1+cd_2)\right]}{\omega^2(1-mL\alpha d_2)}.$$

The mass of the load hanged on the undamaged bar (m = 0) is determined by the formula

$$M = \frac{EF\alpha \text{tg}(\alpha L) - c_1}{\omega^2}.$$

For the bar without incision (m = 0) and at $c \to \infty$ from the equation cos αL = 0 natural frequencies are equal [2] to $\alpha L = (2k-1)\pi/2$ (k = 1, 2, …) or

$$\omega_k = (2k-1)\pi a/2L.$$

To determine m and x_c it is necessary to analyze natural longitudinal vibration frequencies of the bar with incision. Such an investigation was performed for bar's flexural vibrations in [7].

The numerical solution to equation (3) is made for the following parameters of the set: $E = 2 \cdot 10^{11}$ Пa, ρ = 7800 kg/m^3, L = 10 m, F = 0.01 m^2. M = 50 kg, $c_1 = 10^7$ N/m. Sound velocity a = 5063.6 m/s. In this case the first, second and third natural frequencies of the bar without incision are ω_1 = 116 rad/s, ω_2 = 1705 rad/s, ω_3 = 3390 rad/s, respectively. Fig. 2 shows the dependences of parameter m on circular frequencies of the bar's longitudinal vibrations ω_1, ω_2, ω_3 (rad/s) at different x_c / L. These dependences for small values of m are linear. Analysis of the curves shows that it is quite difficult to determine parameter m by the first natural frequency.

Fig. 3 gives the dependences of ratio x_c / L on circular frequencies ω_1, ω_2, ω_3 of longitudinal vibrations at different m. One can see the periodical dependence of circular frequencies of longitudinal vibrations ω_1, ω_2, ω_3 on x_c / L. It is also correct to conclude that determination of the coordinate of incision by the first circular frequency presents difficulty because of accuracy. Big changes in the coordinate of incision and parameter m correspond to insignificant ones in the first circular frequency.

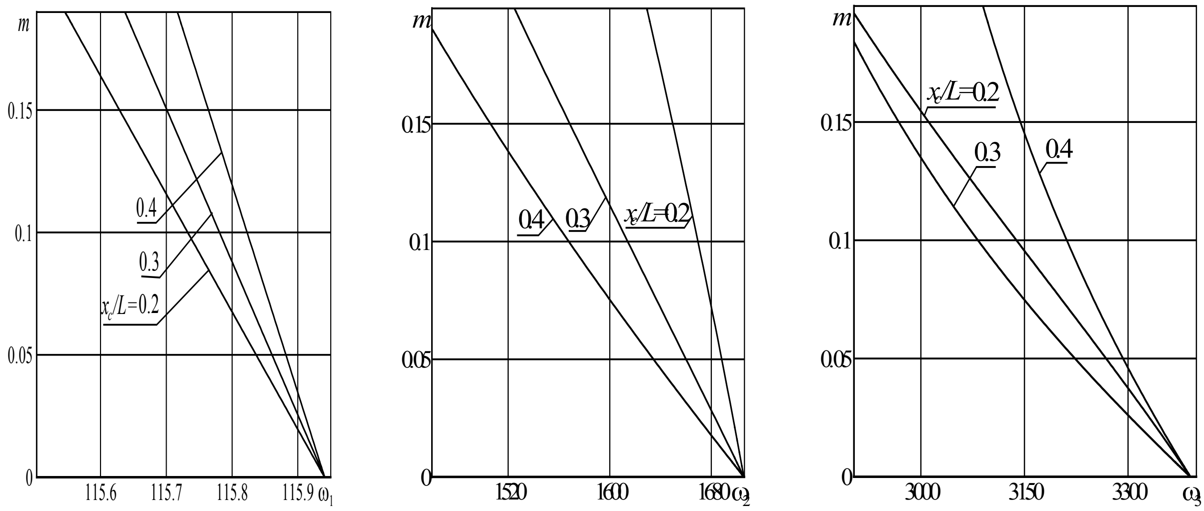

Fig. 2. Dependence of parameter *m* on circular frequencies of bar's longitudinal vibrations $\omega_1, \omega_2, \omega_3$ (rad/s) at different x_c / L.

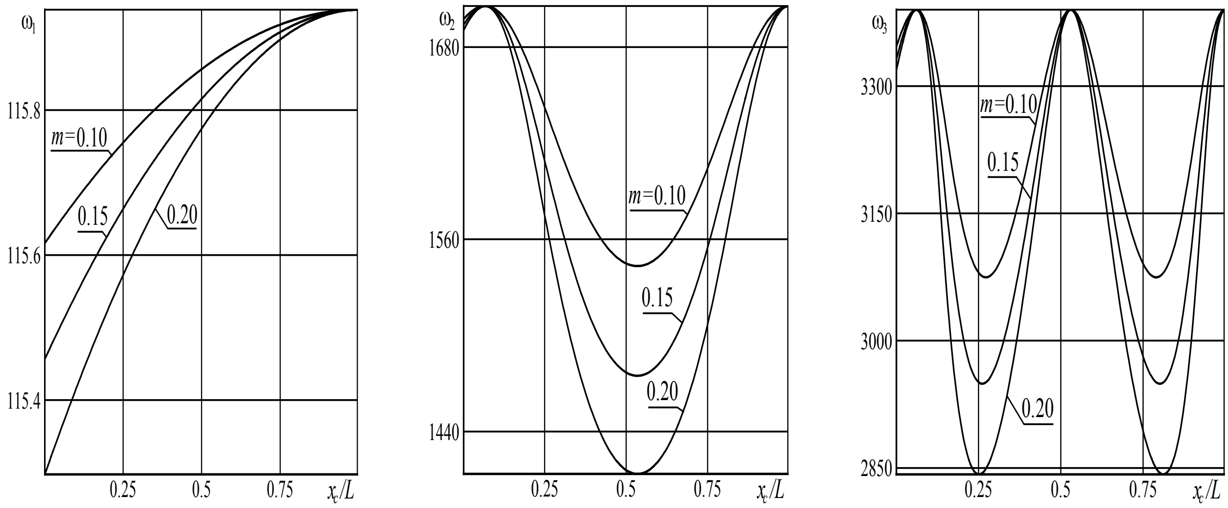

Fig. 3. Dependence of x_c / L ratios on circular frequencies of bar's longitudinal vibrations $\omega_1, \omega_2, \omega_3$ (rad/s) at different *m*.

Fig. 4 shows the dependences of load mass *M* on circular frequencies of the bar's longitudinal vibrations ω_1, ω_2, ω_3 at $m = 0.1$ for different x_c / L. With an increase in circular frequencies of the bar's longitudinal vibrations load mass *M* is enhanced as well. The load mass can be determined by the first frequency.

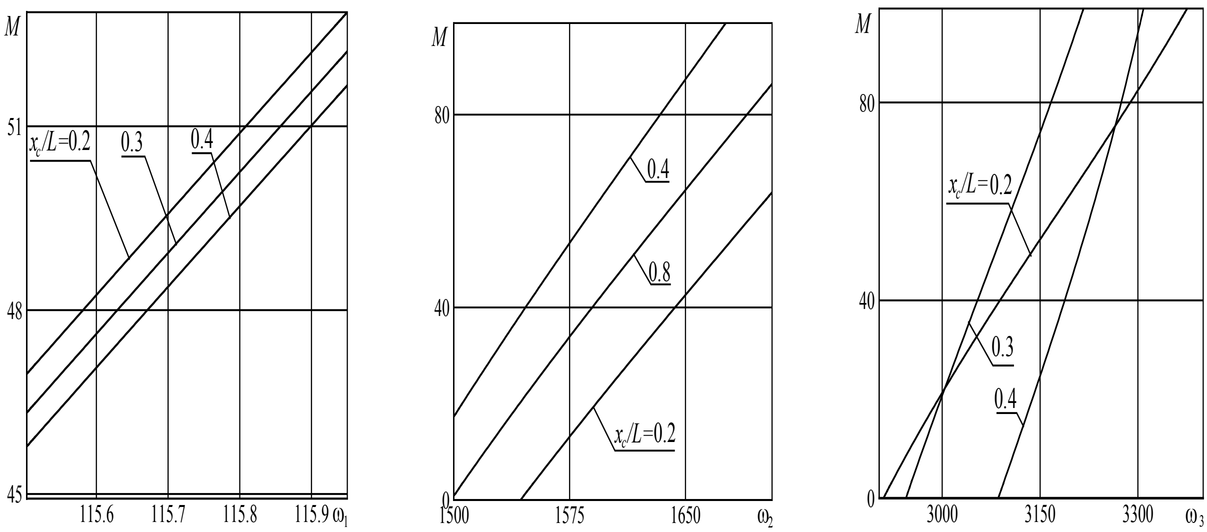

Fig. 4. Dependence of load mass M in kg on circular frequencies of bar's longitudinal vibrations ω_1, ω_2, ω_3 (rad/s) at m = 0.1 for different x_c / L .

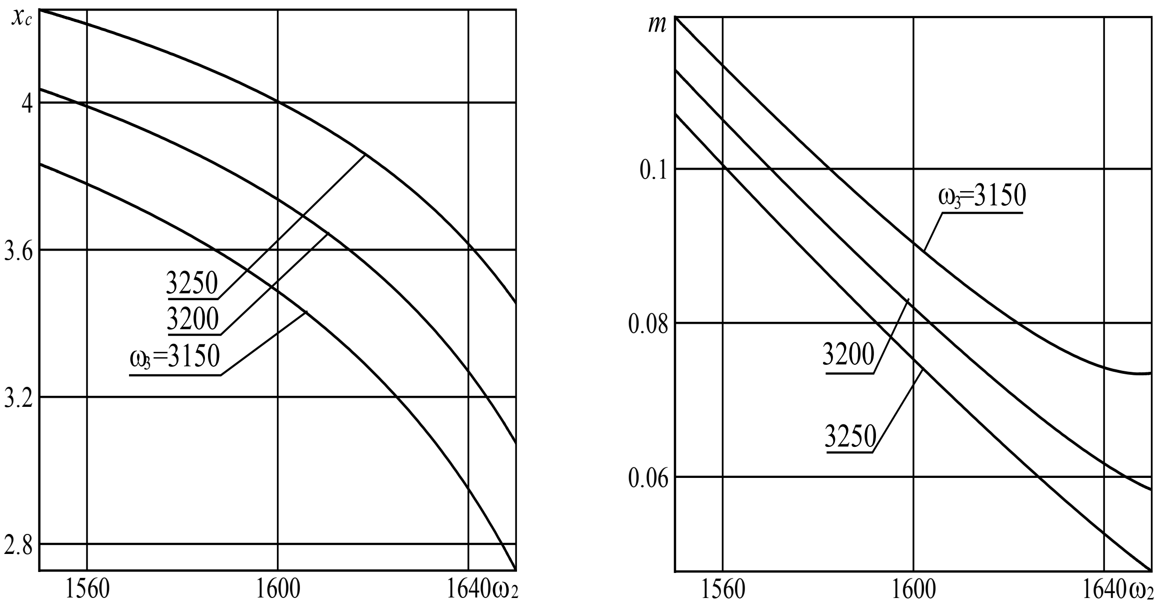

Fig. 5. Dependences of incision coordinate x_c in m and parameter m on circular frequencies of bar's longitudinal vibrations ω_2, ω_3 (rad/s).

If we write the frequency equation for two frequencies of free longitudinal vibrations, using the obtained set of equations we can determine the coordinate of incision x_c and parameter m. Fig. 5 gives the dependences of incision coordinate x_c in m and parameter m on circular frequencies of the bar's longitudinal vibrations ω_2, ω_3 (rad/s).

The performed research shows the possibility of determining the coordinate of incision x_c and parameter m by two natural frequencies of free longitudinal vibrations.

This work has been done under RFFR grant 08-01-97008-r_povolzhie_a.

References

1. Vankov Yu.V., Kazakov R.B., Yakovleva E.R. Natural frequencies of hardware product as informative mark of damage. Electronic Journal "Technical Acoustics", http://ejta.org 2005, 5.
2. Ilgamov M.A. Diagnostics of defects in vertical bar. Proceedings of Institute of Mechanics of Ufa Branch, RAS. Vol. 5. – Ufa: Gilem. 2007. P. 201-211.
3. Vatulian A.O. Inverse problems in deformable body mechanics. Moscow: Fizmatlit. 2007. 224 p.
4. Vatulian A.O., Soluyanov N.O. On determining place and size of cavities in elastic bar // Defectoscopy. 2005. No 9. P. 44-56.
5. Parton V.Z., Morozov E.M. Elastic-plastic fracture mechanics // Moscow: Nauka. 1974. – 450 p.
6. Razyantsev A.O. Vibroacoustic diagnostics of pumping rods in service. Cand. Sci. Thesis. – Ufa: USPTU, 1999. 108 p.
7. Okrushko E.I., Urakseev M.A. Defectoscopy of well pump bars // Moscow: Nedra. 1983. 112 p.