EXPERIMENTAL CHARACTERIZATION OF SENSORS FOR ELASTIC GUIDED WAVES BASED ON 3D LASER VIBROMETRIC MEASUREMENTS AND THE RECIPROCITY RELATION

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1. INTRODUCTION

The development of new Non-destructive testing (NDT) and structural health monitoring (SHM) concepts initiated the development and application of new types of sensors and transducers. So, for structural health monitoring with guided elastic waves a variety of piezoelectric transducers were proposed. Examples are disc, foil and fiber transducers. A reliable characterization of these transducers in its various application conditions is necessary, because the transducer performance will depend on a number of factors. Among them are the way of mounting the transducer (e.g. attached or embedded), the type and thickness of a glue layer and also the structure the transducers are mounted to. Analytical and numerical modeling of the transducer behavior is helpful for characterizing the transducer's behavior, but has also its limitations. It is hard to implement all essential aspects of a transducer and its interaction with the structure. In several cases there is even lack of reliable input parameters to fulfill a model.

A strong substitute and/or supplement of numerical modeling are precise and reliable measurements of the transducer's behavior. The transmission characteristics for sending plate waves can be characterized by non-contact 3D-vibrometer measurements. If the impulse response is measured this way, the signal for an arbitrary electric excitation can be obtained by convolution algorithms.

For the characterization of a transducer as a receiver there is no such easy method at hand, due to the lack of simple reproducible excitation sources. In literature it is often mentioned that the transducer-performance as a receiver equals its performance as a transmitter due to reciprocity. This statement is only true if the corresponding sending and receiving transfer functions are defined appropriate. That subject was discussed in depth for electroacoustic transducers by Primakoff and Foldy [1,2] in early papers. Partly based on that, several papers on self calibration of electroacoustic transducers have been published (for an example see e.g. [3]). The analysis of Foldy and Primakoff is valid for electroacoustic transducers coupled to a fluid. But most ultrasonic transducers coupled to a solid are also included, because they are in contact to the solid via a thin liquid coupling layer. But the Foldy paper is not directly applicable to transducers which are coupled without a liquid layer. For example shear wave NDT-transducers are coupled via a (highly) viscous layer. Further, almost all piezoelectric transducers nowadays used for SHM are glued to the surface. It is difficult to determine well defined receiving transfer functions of guided wave transducers directly by experiments. So, to the best of the authors' knowledge, there are only two papers of one group describing such measurements [4,5].

We will show that a well defined reciprocity relation can be found which is adapted in such a way, that they relate Lamb wave transducer sensitivities based on receiving transfer functions to well defined sending transfer functions in a very general way. That solves the described difficulty concerning the experimental determination of the sensitivity. It is replaced by a measurement of the corresponding sending transfer function. The mentioned relations could be found by more or less straightforward generalization of the work of Foldy and Primakoff. But we find it easier to start with the approach introduced by Auld concerning the investigation of the ultrasonic scattering process [6]. Achenbach reviews the more recent literature and gives a large number of application examples for this formalism [7]. Most of them are focused to find and illustrate solutions of wave propagation problems (partially) including piezoelectric transducers. In [7] there is no example given, explaining how this formulation can be applied for the determination of receiving sensitivities of transducers out of the sending behavior and, to the best of the author’s knowledge, this is also not published elsewhere.

The transducer’s sensitivity can be defined in various ways. In the next chapter various definitions suitable for SHM-Lamb wave transducers are discussed and one is selected for further use in the following sections. In chapter 3 the reciprocity theorem is used to derive a relationship between the transmission transfer function and the transducer’s sensitivity. As an exact equation this relationship needs no further experimentally evaluation. Nevertheless we add an experimentally test for a simple case to convince readers which are not as confident of the theoretical approach. Chapter 5 gives conclusions and possible further applications.

2. HOW TO DEFINE AN APPROPRIATE TRANSDUCER RECEIVING SENSITIVITY

Several ways are known to define a parameter “sensitivity” of a transducer. Which parameter definition should be used depends on the ease of its determination and how close the practical operation of the transducer can be described by it. We will list possible choices and discuss them with focus on transducers for guided ultrasonic waves.

In general, the term sensitivity of an electromechanical transducer is understood as the ratio of some generated electrical quantity caused by an external mechanical excitation or an incoming wave of a given amplitude. These quantities have to be selected such, that a linear relationship between them exists.
The electrical quantity is usually chosen to be the current through or the voltage over a terminating resistor with real impedance. We choose the voltage U for open termination conditions. On the mechanical/acoustical excitation side the choice is more complex. Fist it is necessary to decide whether the transducer itself is described or the system of the transducer attached to a given propagation medium. The first choice has the advantage, that a thus defined sensitivity is a property of the sensor alone. But we do not see an easy way to derive conclusions for the sensitivity of the coupled system “transducer attached to the propagation medium”, which is the only sensitivity relevant for practical applications[1]. Consequently, we select the second alternative and concentrate on the sensitivity of the transducer glued to the “object”. For Lamb wave transducers and in the simplest case the object would be an infinite extended plate of given thickness d and known material properties. For the sake of simplicity we think about an isotropic and homogeneous plate material, so it would be characterized by its longitudinal and shear wave velocities and its density only, but the approach is not restricted to that limitation.

Defining the physical quantity, which excites the system we can choose (among others) between:

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[1] In SHM the situation it is a bit more difficult as in acoustics or in NDT applications of ultrasound. In the latter cases the propagation medium can be described usually more or less as infinite media which is characterized simply by its acoustic impedance. Therefore the derivation of the sensitivity of the coupled system out of that of the bare sensor is possible what is not valid in the SHM case.

a) an incoming monochromatic plane wave ("plane" is considered in the 2D coordinate system of the plate),
b) an incoming wave train (e.g. pulse),
c) a wave generated by a point source of some kind; either point force on the surface or given point displacement at the surface, or
d) a wave excited by a standard transducer in a given position relative to the receiver; the transducer itself is excited by a defined electric pulse.

A sensitivity defined with respect to an incoming monochromatic plane wave (a) is a very valuable quantity. For a plate there will be a series of such sensitivities indexed by the Lamb wave modes. Additionally, the sensitivity for each mode will depend on the incident wave vector orientation with respect to the transducer orientation. Based on this, the transducer response to an arbitrary monochromatic incoming plate wave can be obtained by a simple integration over the angular distribution and a summation over the Lamb wave modes. All monochromatic sensitivities are hard to obtain experimentally, because they essentially involve wave propagation in an infinite extended plate.

The choice of the pulse option (b) avoids the difficulty of choice (a) concerning the infinite plate dimension. But the source of the pulse and its spectral composition must be specified and also experimentally realized. Such specific choice of a certain pulse shape is a serious limitation.
The definition of a transducer sensitivity based on an excitation according to (d) comes essentially down to the definition of a transfer function in a pitch catch configuration where the “pitcher” is a standard transducer. This seems to be not very general.
So, option (c) remains. We prefer a point force to a point displacement. Additionally, the position of the excitation must be specified, otherwise the approach is rather general. Especially, if the sensor sensitivity is known in dependence of the excitation position and for all excitation force directions nearly all transducer configurations can be realized by a simple integration over the aperture of the transducer. Furthermore, scattering fields can also be described as superposition of normal forces.

3. APPLICATION OF THE RECIPROCITY RELATION TO RELATE SENDING AND RECEIVING SENSITIVITIES.

To find the formulation of the reciprocity between sending and receiving properties of a sensor in combination with a propagation object, we start with the general formulation of the reciprocity for a piezoelectric solid as formulated by Auld (1979). We follow mainly the formulation of Achenbach (2003).

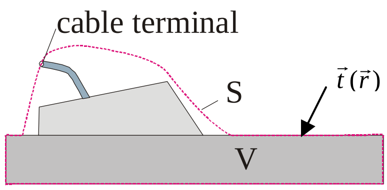

Figure 1: Transducer coupled to a solid; the integration volume V has the boundary S which is indicated by a dotted (red) line. The boundary crosses the electrical coaxial cable.

The following derivation is valid for any volume V which contains the transducer completely. We consider one set of electromechanical field variables within the same volume V under two different conditions "A" and "B". Each field is characterized by the stress tensor $\vec{\vec{\sigma}}$, the mechanical particle velocity field $\vec{v}$, the electric field $\vec{E}$ and the magnetic field $\vec{H}$. External sources are fields of the force density $\vec{\boldsymbol{f}}$ and the current density $\vec{\boldsymbol{j}}$. Characterizing both sets of filed variables by the corresponding upper index "A" and "B", respectively, it can be proved [6] that they fulfill a reciprocity theorem of the form

$$\int\limits_{S=\partial V}\left(\vec{t}^{\,A}\vec{v}^{\,B}-\vec{t}^{\,B}\vec{v}^{\,A}\right)dS+\int\limits_{S}\left(\vec{E}^{\,B}\times\vec{H}^{\,A}-\vec{E}^{\,A}\times\vec{H}^{\,B}\right)\vec{n}dS=\int\limits_{V}\left(\vec{f}^{\,B}\vec{v}^{\,A}-\vec{f}^{\,A}\vec{v}^{\,B}+\vec{j}^{\,B}\vec{E}^{\,A}-\vec{j}^{\,A}\vec{E}^{\,B}\right)dV \quad (1)$$

To specify the two conditions we define "A" as operating under open electrical conditions meaning no source on the electrical side $\boldsymbol{I}^{\boldsymbol{A}}=0$ is present. In the second case "B" the surface is free of any mechanical traction $\vec{t}^{\,B}=0$. Respectively, no source is active at the mechanical side.

Obviously, equation (1) is rather applicable to direct measurements. To keep the scope of this paper, the derivation of the final terms is described in a generally understandable manner. The detailed derivation will be published elsewhere. Basically, linearity of the whole system is supposed. The surface traction $\vec{t}$ is introduced as product of the stress $\vec{\vec{\sigma}}$ and the surface normal vector $\vec{n}$. Hence, the considered forces are perpendicular to the surface. Further, all quantities are considered as time harmonic, so (1) is an equation in the frequency range. Now, we assume that the only source terms are the electrical sources at the cable end in the transmitting case "B" and the introduced force in the receiving case "A" of the piezoelectric transducer. Concerning the field of surface traction $\boldsymbol{t}^{\boldsymbol{A}}$ a point like excitation is introduced at the location $\tilde{r}_0$. Merging these components into equation (1) and incorporating arising mathematical simplifications equation (2) follows as

$$\frac{\boldsymbol{U}^{\boldsymbol{A}}}{\boldsymbol{t}^{\boldsymbol{A}}}=\frac{\boldsymbol{v}^{\boldsymbol{B}}}{\boldsymbol{I}^{\boldsymbol{B}}} \quad (2)$$

meaning the ratio of the voltage $\boldsymbol{U}^{\boldsymbol{A}}$ generated under open electrical conditions ($\boldsymbol{I}^{\boldsymbol{A}}=0$) due to a point surface traction $\boldsymbol{t}^{\boldsymbol{A}}$ at the point $\tilde{r}_0$ under otherwise stress free surface conditions equals the ratio of the surface velocity v^B at the same point $\tilde{r}_0$ exited by an current $\boldsymbol{I}^{\boldsymbol{B}}$ under traction free conditions to $\boldsymbol{I}^{\boldsymbol{B}}$ wherein v^B is the surface velocity component in the direction of the surface traction, that is

$$\boldsymbol{v}^{\boldsymbol{B}}=\vec{v}^{\,B}\frac{\vec{t}^{\,A}}{\left|\vec{t}^{\,A}\right|}$$

Equation (2) is the desired relation between a well defined sending transfer function and a corresponding sensitivity as sensor. Of course, (2) is an equation in the frequency range. Introducing for both sides of (2) a transfer-function $tr(\omega)$ we can equivalently also write

$$\boldsymbol{U}^{\boldsymbol{A}}(\omega)=\boldsymbol{tr}^{1}(\omega)\boldsymbol{t}^{\boldsymbol{A}}(\omega) \qquad \boldsymbol{v}^{\boldsymbol{B}}(\omega)=\boldsymbol{tr}^{2}(\omega)\boldsymbol{I}^{\boldsymbol{B}}(\omega) \quad (3)$$

and (2) corresponds to

$$\boldsymbol{tr}^{1}(\omega)=\boldsymbol{tr}^{2}(\omega) \quad (4)$$

Due to simplicity, we use identical symbols for the functions in the time and frequency domain. Which domain is meant will be clear by the context and the argument. By inverse Fourier transformation in the time domain and using the convolution formalism we get

$$U^A(t) = \int tr^1(t-\tau)t^A(\tau)d\tau \tag{5}$$

$$v^B(t) = \int tr^2(t-\tau)I^B(\tau)d\tau \tag{6}$$

with

$$tr^1(\tau) = tr^2(\tau) \tag{7}$$

It has to be kept in mind that both equations refer to the same point $\tilde{r}_0$ at the surface and the same surface unit vector. Here the surface traction $\vec{t}$ is acting in the same direction the surface velocity $\vec{v}$ is measured.

The proposed strategy for the determination of the sensitivity of a given transducer as considered here is:

(1) Determination of the pulse response $tr^2(\tau)$ out of equation (6); This can be done by exiting the system with a current pulse $I(t) = I_0\delta(t-t_0)$ and measuring the velocity $v^B(t)$ at $\tilde{r}_0$; Hence, the pulse response $tr^2(\tau)$ equals $v^B(\tau)/I_0$. If the current can't be mad sufficiently pulse like a second deconvolution must be applied to the signal.

(2) Using $tr^1(\tau) = tr^2(\tau)$ in equation (5) to obtain the voltage U for a given force excitation

Today contactless laser vibrometer measurements of surface velocities are possible for all 3D components and in an automated scanning way, so corresponding vector fields of transfer functions can be measured. In principle, this allows to determine the response of the transducer to an arbitrary vector field of surface tractions.

4. EXPERIMENTAL VERIFICATION

The reciprocity relation (2) is only valid if the requirements in (1) and the additional assumptions made in chapter 3 are fulfilled. As already mentioned only nonlinear and furthermore hysteretic behavior are excluded by these requirements. While actually no experimental verification is really available, we will give data from a carefully designed experiment. Here it is shown that the relation is applicable. Additionally, the deviations occurring within relation (2) after applying to measured data reveals hints for limitations within the experiment. This verification will be done for the special case that the vector $\vec{p} = \vec{t}/|\vec{t}|$ is the surface normal vector $\vec{n}$. So both, the excitation force and the detected velocity component will act normal to the surface.

The transducer used for the experiment is a circular piezoelectric disc of 400 µm thickness and a diameter of 10 mm. The disc was glued onto a plane aluminum plate with dimensions of 1000 x 1000 x 2 mm³. The distance of the disc center to the detection/ excitation point $\tilde{r}_0$ is 150 mm.

The reciprocity relation will be tested in the time domain by proving that (5) and (6) are connected via (7). The easiest way would be to use a delta function excitation in both cases to reduce the integrals to its kernels. We will go a slightly modified version as described below. Due to the differing bandwidths the used devices have, the signals were limited in frequency to build a concerted region of validity.

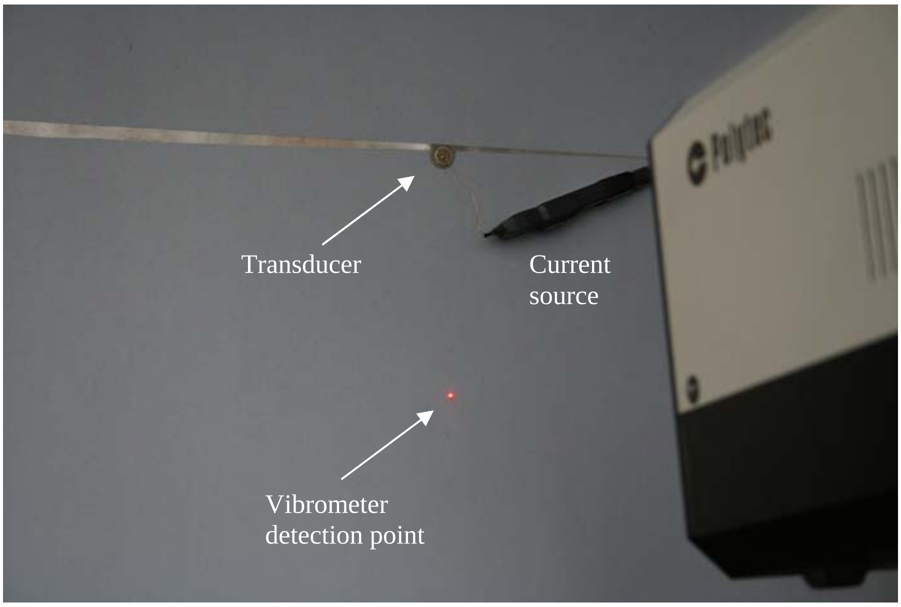

Figure 2: Image of the set up to measure the out of plane velocity component by laser-vibrometry.

Transmitting experiment:

To determine of the transmitting sensitivity the transducer was excited by a current pulse I. It was generated by a step function of the transducer voltage and measured by the voltage drop ΔU over a small resistor R_p as $I = \Delta U / R_p$ as shown in figure 4. Here, the transducer's capacity smears the initially sharp pulse and causes oscillations within the signal. The normal component of the displacement velocity vector was measured at the point $\tilde{r}_0$ by a Laser Doppler vibrometer (figure 2).

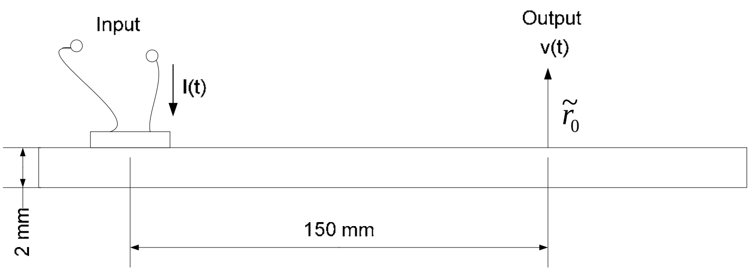

Figure 3: Experimental set up in the case when the transducer acts as transmitter. The current I is send and the velocity v is measured

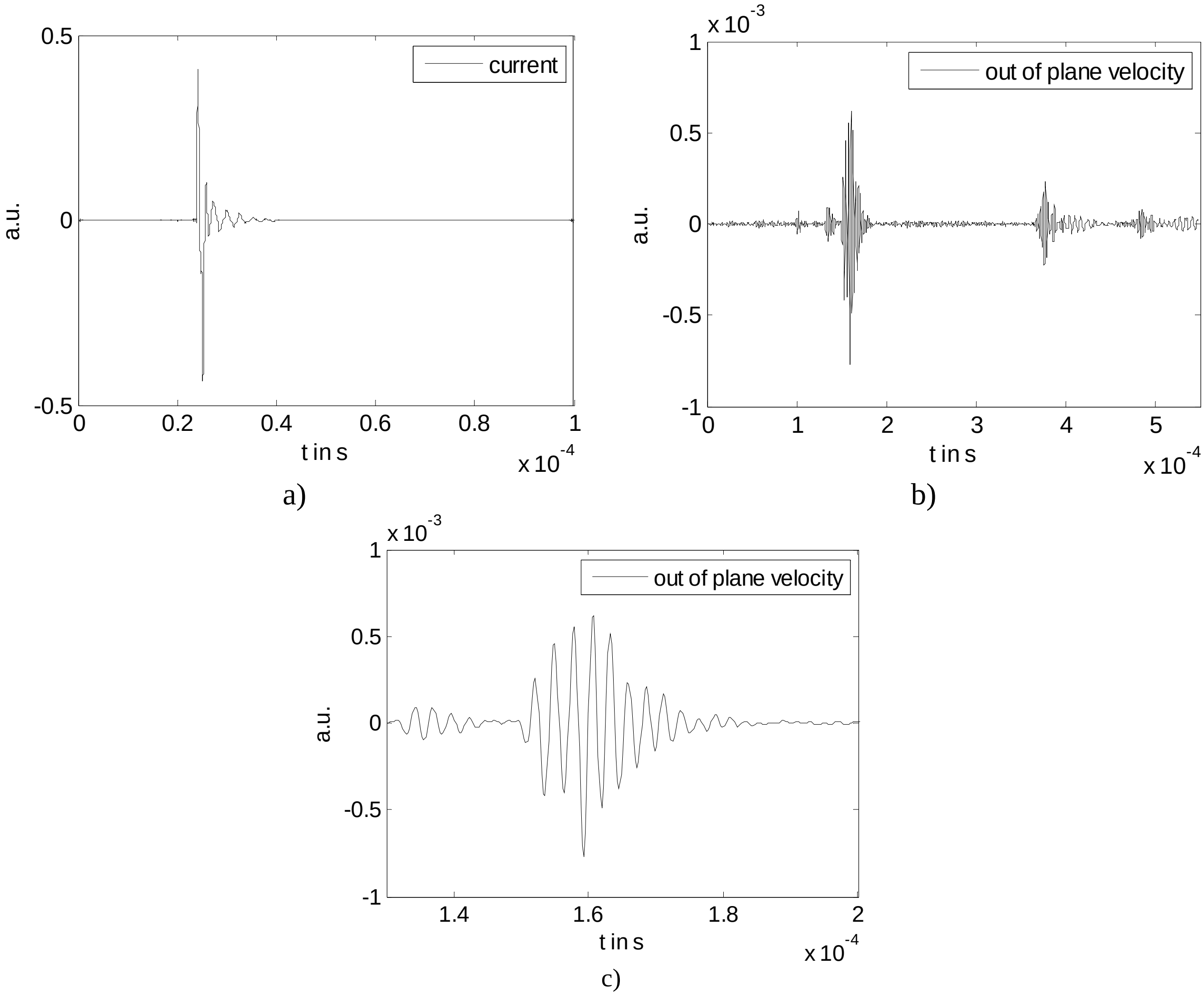

Figure 4: Current I through the transducer after voltage pulse excitation (a), measured out of plane velocity (b), and first 2 arriving wave modes as part of out of plane velocity taken for further calculations (c)

As already mentioned the current *I* was filtered by a low pass with 5 MHz cut-off frequency, for the velocity signal (figure 4). The transducer signal (figure 5) was sent to a band-pass filter operating at 100 kHz to 500 kHz.

Receiving experiment:
To evaluate the receiving case, one has to measure the voltage response of the system to a surface force excitation. Because it is difficult to generate a pulse-like surface force, a step like force excitation by a pencil lead break is used instead. This method is widely used in acoustic emission testing for coupling control and as a standard excitation source [9], [10]. The pencil lead break produces a reproducible step like force release with rise/fall times of about 1 µs which corresponds to an upper frequency limit of 1 MHz. This forth release can be described with the Heaviside step function H as

$$\boldsymbol{t}^{A}(\boldsymbol{t}) = -\boldsymbol{t}^{A}{}_{0}\boldsymbol{H}(-\boldsymbol{t}) \qquad (8)$$

Setting this into (5) and differentiating with respect to t gives the simple relation

$$\frac{dU^A(t)}{dt} = t^A{}_0 tr^1(t) \qquad (9)$$

Unfortunately, it is hardly possible to get the magnitude $t^A{}_0$ of this excitation reproducibly. However, the evaluated signals fit well as shown later and the reciprocity is proved up to a common factor.

Figure 4: Experimental set up in the case of the transducer acting as receiver. A normal force is excited and induces a voltage output at the open electrical contacts of the transducer.

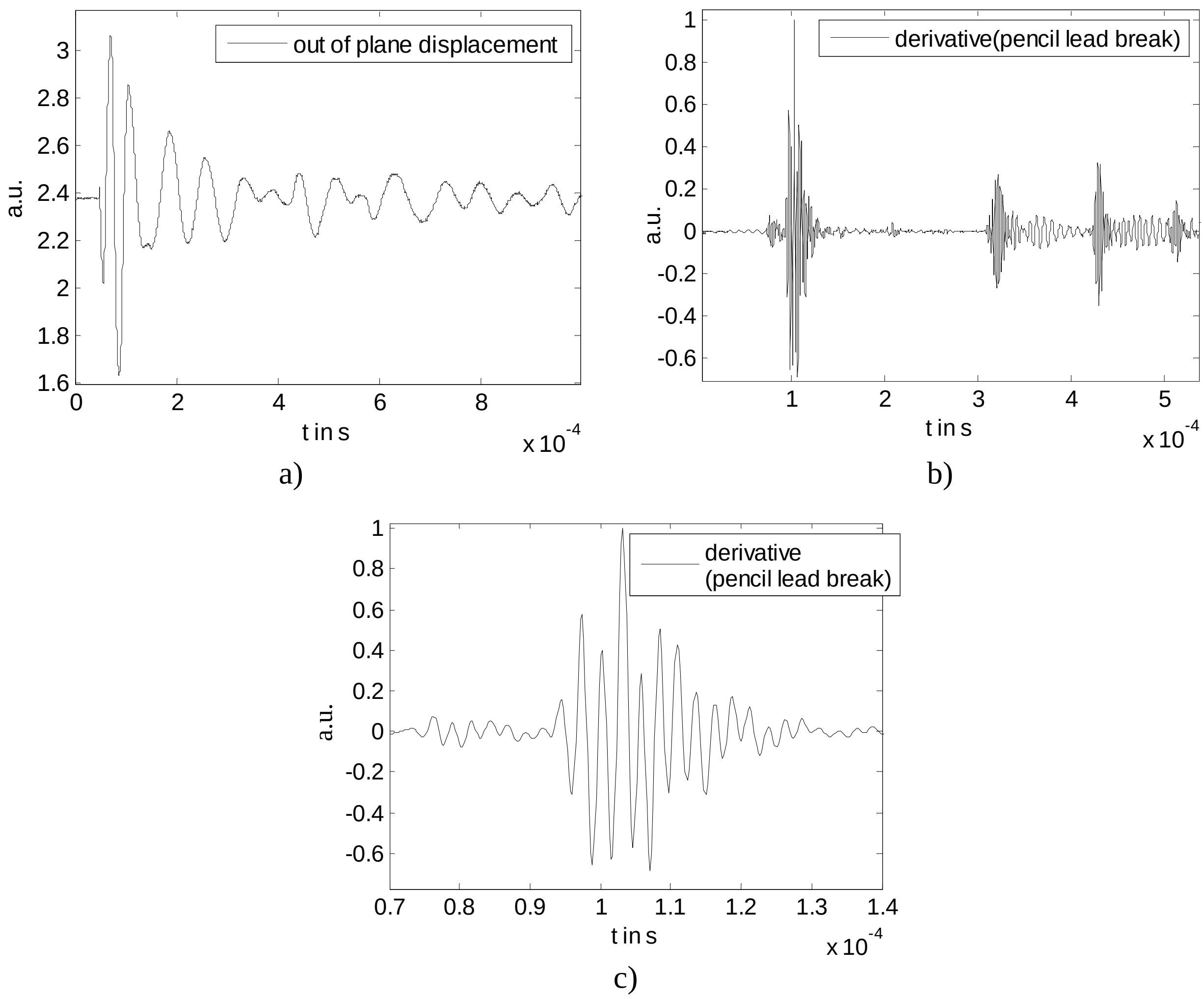

Figure 5: Out of plane displacement measured by laser vibrometry at the location of the pencil lead break (a), derivation of the measured transducer signal as the answer to a surface force pulse excitation (b), first 2 arriving wave modes as part of the transducer signal taken for further calculations (c);

Application to reciprocity relation
After the acquisition of these four different time signals they can now be used to solve equation 2. If the proclaimed reciprocity relation holds, (7) must be valid with (5) and (6). We prove that indirectly by comparing the measured velocity $\boldsymbol{v}^{\boldsymbol{B}}(\boldsymbol{t})$ with the signal calculated as

$$\boldsymbol{v}^{\boldsymbol{B},1}(\boldsymbol{t}) = \frac{1}{\boldsymbol{t}^{\boldsymbol{A}}{}_0} \int \frac{\boldsymbol{dU}^{\boldsymbol{A}}(\boldsymbol{t}-\tau)}{\boldsymbol{dt}} \boldsymbol{I}^{\boldsymbol{B}}(\tau)\boldsymbol{d}\tau = \int \boldsymbol{tr}^1(\boldsymbol{t}-\tau)\boldsymbol{I}^{\boldsymbol{B}}(\tau)\boldsymbol{d}\tau \qquad (10)$$

with $\boldsymbol{tr}^1$ taken from (9).

Because we can express the measured velocity $\boldsymbol{v}^{\boldsymbol{B}}(\boldsymbol{t})$ by

$$\boldsymbol{v}^{\boldsymbol{B},2}(\boldsymbol{t}) = \int \boldsymbol{tr}^2(\boldsymbol{t}-\tau)\boldsymbol{I}^{\boldsymbol{B}}(\tau)\boldsymbol{d}\tau \qquad (6)$$

both signals should coincide if equation (7) ($\boldsymbol{tr}^1(\tau) = \boldsymbol{tr}^2(\tau)$) is valid, that is if the reciprocity holds. In Figure 6 both normalized signals are plotted against time and show indeed a good agreement.

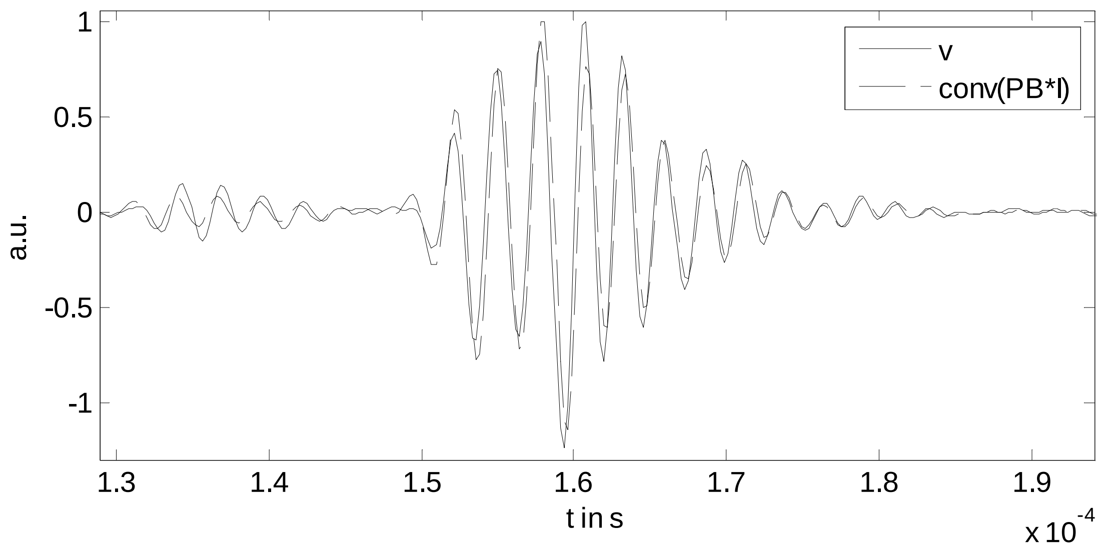

Figure 6: Comparison of the measured surface normal velocity $\boldsymbol{v}^{\boldsymbol{B}}(\boldsymbol{t})$ and quantity calculated according to equation (10); the agreement proves the validity of the reciprocity relation.

5. CONCLUSION

Within this paper a basic application of the reciprocity relation exerted to surface waves was demonstrated. Based on a well-designed experiment and carefully consideration of proper chosen input signals a good agreement between measurements and reciprocity related derivations can be achieved. The incorporation of laser-vibrometry enables new possibilities for characterizing sensors and actors used in NDT and SHM systems. Besides the evaluation of the valid scope of the presented relation it is also possible to characterize a transducer's sensitivity and its angular dependencies. Further studies will concentrate on the magnitudes of the determined signals to complete the model. Summing up and as shown in figure 6, reciprocity applied to surface waves is feasible and benefits a SHM sensor system's design.

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