VIBRATION MEASUREMENT UNCERTAINTY AND RELIABILITY DIAGNOSTICS RESULTS IN ROTATING SYSTEMS

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1. Introduction

The rotating machinery technical state can be described with various parameters. There are five main non-destructive control methods [1, 2], usually applied during machinery state monitoring: vibration monitoring, termography, tribology, ultrasonics and visual inspection. Vibration monitoring and diagnostics is the universal method, used for different equipment observation, it allows determining and evaluating the problems and defects emerging in any stage of equipment exploitation.

As vibration monitoring uses vibration measurement for diagnostics, it is important to evaluate the quality of measurement. Vibration measurement uncertainty describes this quality and is usually evaluated using procedure provided in the "Guide to the Expression of Uncertainty in measurement" – GUM [3]). Here measurement uncertainty is defined as parameter related to the measurement result and variance of values which can be reasonably attached to the measurand. The measurement uncertainty consists of these components:

- uncertainty due to measurement mean error;
- uncertainty due to environment factors influencing measurement result;
- uncertainty of the standard;
- uncertainty due to assumptions of the measurement methods.

The standard measurement uncertainty evaluation procedure [3, 4]:

- evaluate every effect x_i, which significantly influences the measurement result y;
- evaluate each uncertainty component, which significanly contributes to the uncertainty of measurement, described by a standard deviation $u_i(x_i)$ (standard uncertainty);
- determine the combined standard uncertainty u_c, by combining the individual standard uncertainties (and covariances) according to the formula:

$$u_c(y) = \sqrt{\sum_{i=1}^{N}\left(\frac{\partial f}{\partial x}\right)^2 u^2(x_i) + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\frac{\partial f}{\partial x_i}\frac{\partial f}{\partial x_j}u(x_i, x_j)}\ , \qquad (1)$$

here $u(x_i, x_j)$ is estimated covariance associated with measurands x_i and x_j; N – number of parameters;

- determine the expanded uncertainty U by multiplying u_c by a coverage factor k:

$$U = ku_c. \qquad (2)$$

The value of k depends on desired level of confidence to be associated with the uncertainty interval. A value of k is in the range 2 to 3. When there is enough measurements made, then normal distribution describes combined uncertainty and coverage factor $k = 2$. If measurement uncertainty is evaluated for a small set of measurements, then $k \neq 2$ and usually is stated separately next to the measurement result.

Usually the GUM recommends using two different ways to evaluate uncertainty components. These methods are called „A" and „B": „A" method uses statistical methods for uncertainty component evaluation – the specific data set is analyzed, its statistical parameters are evaluated, and using "B" method other data than statistical is used, although the man result is statistical evaluation.

Measurements are used in diagnostics have an important place, particularly in differential diagnostics. The received data and their limit values form decision making rules. The result of the rule application may correspond to the real state (diagnosis), but also measurement uncertainty may influence this result and in this case it might be different from real situation (diagnosis). This requires investigating uncertainty of measurements, used in this diagnosis and observing its influence to the results of decision making. The paper analyses correlation between vibration measurement uncertainty in vibration monitoring and diagnostics systems and diagnostics false results.

2. Vibration monitoring uncertainty

Vibration monitoring and diagnostics systems (monitoring can be permanent or periodic) consist of two subsystems – measurement and diagnostics. The reliability of those two subsystems influences common realibility of monitoring and diagnostic system and probability of false diagnosis.
Vibration measurement influence factors according to their nature can be distinguished to:

- measurement mean error components;
- environmental influence components;
- time components.

If the variation of the process is significantly big, then statistical uncertainty component might be big and significant while calculating general uncertainty evaluation. Although in case of stationary monitoring system the measurand is usually stable. As we do assumption that vibration measurement result set in case of permanent monitoring is stationary process, then $\sigma^2 = \frac{\sum_{j=1}^{N}\left(x_j - \bar{x}\right)^2}{N-1}$ becomes a fixed value and then systematic measurement error (statistical uncertainty component) may be expressed as:

$$u_{stat} = \sqrt{\frac{\sum_{j=1}^{N}\left(x_j - \bar{x}\right)^2}{N-1}} \cdot \frac{1}{\sqrt{N}} \approx \frac{\sqrt{\sum_{j=1}^{N}(x_j - \bar{x})^2}}{N} \tag{3}$$

Knowing the limit when the uncertainty component might be disposed as unsignificant, let's mark this ratio as b, thus we can calculate the necessary data volume in order to get insignificant uncertainty component.

$$\text{If } \frac{|\Delta x|}{\sqrt{N}} \le b \text{ , then } \sqrt{N} \ge \frac{|\Delta x|}{b} \text{ or } |N| \ge \left(\frac{\Delta x}{b}\right)^2 \tag{4}$$

In case of periodic monitoring the influence of statistical uncertainty is emphasized, in case of permanent monitoring – systematic error is more important.
The uncertainty of permanently installed monitoring system is calculated [5] according the formula:

$$U_{SVMS} = 2 \cdot \sqrt{u_{stat}^2 + u_{mat}^2 + u_{\hat{K}}^2 + u_{K_{ss}}^2 + u_T^2 + u_H^2 + 2u_T u_H r(T,H) + u_{tr}^2} \tag{5}$$

Here:
u_{stat} - statistical contribution, or random error, calculated using measurement data;
u_{mat} - contribution due to amplitude-frequency characteristic unevenness;
$u_{\hat{K}}$ - error of transducer transformation coefficient;
$u_{K_{ss}}$ - error of transverse transformation coefficient;
u_T – contribution of temperature T;
u_H – contribution of humidity H;
$r(T,H)$ - correlation between temperature and humidity.
The uncertainty model of periodic vibration monitoring system could be calculated according to the formula:

$$U_{NVMS} = k \cdot \sqrt{u_{stat}^2 + u_{kal}^2 + u_T^2 + u_H^2 + 2u_T u_H r(T,H) + u_{laik}^2 + u_{op}^2} \tag{6}$$

here u_{op} is error due to operator, as the measurements usually are performed manually by operator.
The main difference between permanent and periodic monitoring systems is that here the main roles are played by different error types. In stationary system the measurements are performed constantly and the statistical uncertainty is very small, as the number of N is large, but the transducers are calibrated periodically, for example, once per year. Influence factors, determined during calibration,

still affect the measurement procedure if it is performed in environmental conditions other than of calibration. So the systematic error here plays the main role rather than the random. In case of periodic monitoring system, the transducer calibration is usually performed before every measurement set. But in contrary, as the measurements are rarer, their variance is bigger and the volume of data set is very small – the random error dominates in uncertainty value.

3. Analysis of measurement uncertainty influence to diagnostic parameters.

According to the definition, the evaluation of reliability is a quantitative evaluation of the product or system. For such evaluation usually mathematical modelling is used, directly applied results of the tests, etc.

Data sets in monitoring systems are usually large, so the distribution of the diagnostic parameter *x* might be described as:

$$f(x/D_i) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\bar{x}_1)^2}{2\sigma^2}} \tag{7}$$

Here:

$\bar{x}_1$ - mean value of the data set, further is also denoted as *m*;

σ - standard deviation of the data set;

D_i – state of the system (D_1 – stable state; D_2 – defective state).

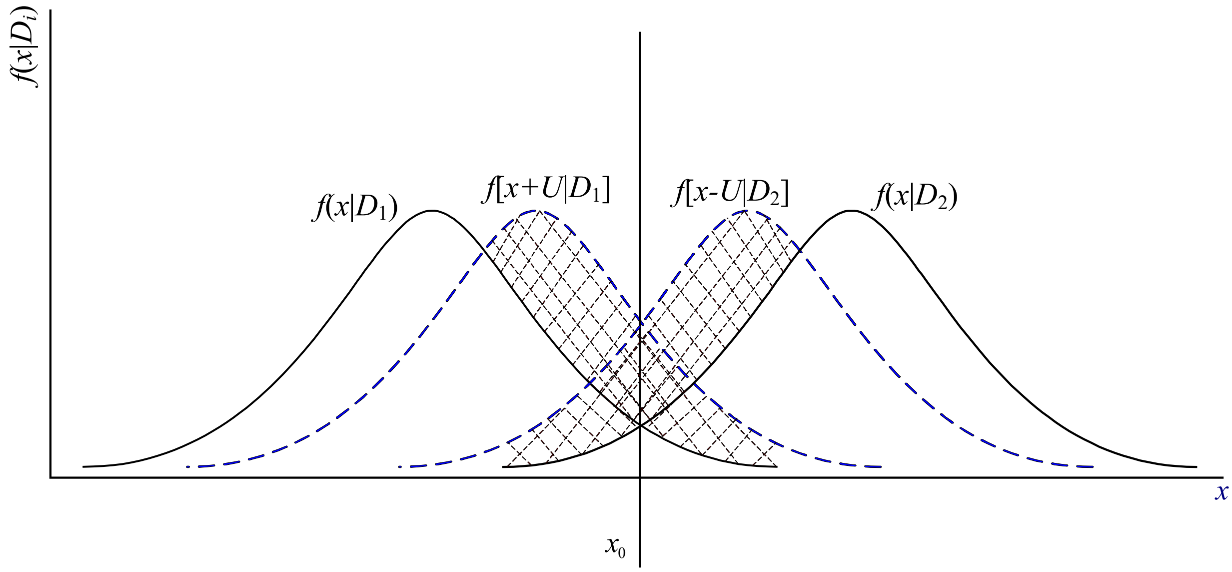

Fig. 1. Uncertainty influence to decision making probability

Possible uncertainty influence to decision making probability is depicted in Fig. 1. The area of the first distribution which falls outside the x_0 line is the probability to make a false decision [6]. When the uncertainty estimate *U* is added to the measurement result, the graph moves to the right. So, the area outside the x_0 line changes – it becomes bigger, in other words, the probability of making false decision increases. The same principles can be applied to the second curve in solid line; therefore we will analyze only the first case.

The probability to make a false decision (Fig. 1) can be expressed by formula:

$$P(H_{21}) = P_1 \int_{x_0}^{\infty} f(x/D_1)dx \tag{8}$$

Here P_1 is constant. Influence of measurement uncertainty to decision making is described as:

$$f(\bar{x},\sigma,U,x_0)=\frac{1}{\sigma\sqrt{2\pi}}\int_{x_0}^{3\cdot\sigma}e^{-\frac{(x-(\bar{x}+U))^2}{2\sigma^2}}dx-\frac{1}{\sigma\sqrt{2\pi}}\int_{x_0}^{3\cdot\sigma}e^{-\frac{(x-\bar{x})^2}{2\sigma^2}}dx$$
$$=\frac{1}{\sigma\sqrt{2\pi}}\left(\int_{x_0}^{3\cdot\sigma}e^{-\frac{(x-(\bar{x}+U))^2}{2\sigma^2}}dx-\int_{x_0}^{3\cdot\sigma}e^{-\frac{(x-\bar{x})^2}{2\sigma^2}}dx\right) \tag{9}$$

The function $f(\bar{x},\sigma,U,x_0)$ describes the change of false decision probability when the area is constrained by limit value x_0 and 3σ. The function will depend on the mean, variance, limit value and uncertainty.

During the analysis the maximum uncertainty was chosen to be half of the mean value $U\le\frac{m}{2}$.

The results showed that then there is a clear dependence on the ratio of the mean and uncertainty $\frac{m}{U}$ – the bigger is uncertainty, the bigger fail probability we can expect. We suggest using this ratio as a parameter to calculate a decision making probability change.

Applying this to different vibration monitoring systems, we should notice that in permanent vibration monitoring systems the variance of the normal distribution will be small enough due to large data sets, so in critical cases, the uncertainty influence will show the effect immediately. In case of periodic monitoring system, the data sets are small and variance is much bigger. Due to that, the uncertainty might have a big influence to decision making with portable systems having initial data. The data set might be expanded using the particular methodology, while transformation function will enlarge data set, in this way reducing the statistical uncertainty component. Using this method additional uncertainty contribution should be added, which calculates the impact of the increase of the set to measurement reliability. In the other hand, this contribution has a less impact than the difference between the primal statistical uncertainties and uncertainties calculated after the transformation. As example of the real monitoring and diagnostic system was used vibration monitoring system, installed in Kaunas hydroelectric power station. This data was analysed to evaluate the significance of the threshold $b_i=\frac{|\Delta x_i|}{\sqrt{N}}$.

The values of threshold b_i in stationary monitoring and diagnostics system

Table 1

Threshold	b_1	b_2	b_7	b_8	b_9	b_{10}	b_{11}	b_{12}
Value	$5{,}68\cdot10^{-5}$	0,00136	$1{,}728\cdot10^{-4}$	$2{,}48\cdot10^{-4}$	$8{,}75\cdot10^{-4}$	$6\cdot10^{-4}$	0,0044	0,0036

The noise component in uncertainty model was evaluated using statistical data and $u_{tr}=1{,}568\cdot10^{-3}$ mm/s

This shows that it is unsignificant and does not have any influence to the evaluate of uncertainty. After calculating the parameters included into uncertainty expression, the result of the permanent vibration monitoring system is $U_{SVMS}=0{,}053$ mm/s

The example of the periodic monitoring system was the measurement made for the leak cleaning equipment compressor. According to the analyzed results the conclusions about the periodic system data features were made. In order to evaluate the effect of data volume change to the threshold *b* the graph was made in Fig. 2.

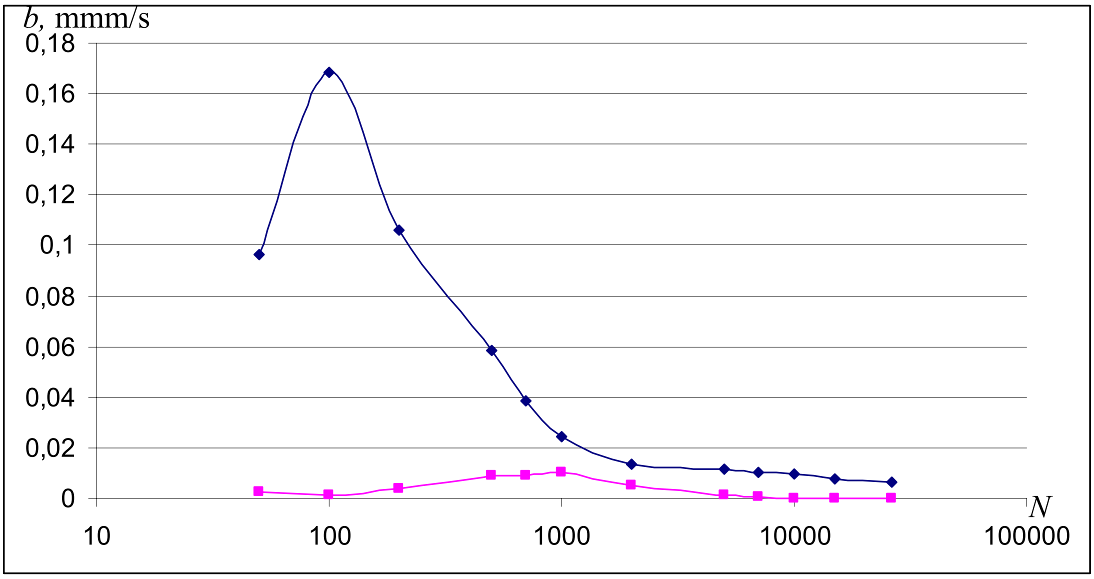

Fig. 2. The dependence of threshold b_i to measurement amount N

Conclusions

1. In the permanent vibration monitoring and diagnostic system due to large quantity of the data the statistical uncertainty component is diminishing and the expanded uncertainty is influenced only by environmental and instrumental uncertainty components. Therefore each similar measurement channel which measures the same quantity will have the same uncertainty evaluate.
2. The variable b was evaluated which allows evaluating the significance of measurement uncertainty statistical component. This variable b depends on the amount of measurement data. When the quantity of this data increases, the variable b decreases. The rate of decreasing depends on the dispersion of the data. If the dispersion is large, then the decrease rate might be insufficient and then the measurement uncertainty is different in each measurement channel as the statistical uncertainty component is quite large.
3. The criterion b is based on the ratio of measurement data average and uncertainty as it evaluates the significance of uncertainty influence to false decision making. This influence is significant only in case when this ratio is equal or bigger than 10 mm/s.

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