MODELLING OF DYNAMICS OF HETEROGENEOUS SYSTEMS AND STATE IDENTIFICATION

Vitalijus VOLKOVAS

KAUNAS UNIVERSITY OF TECHNOLOGY
Technological Systems Diagnostic Institute
Lithuania

Abstract. The condition monitoring and diagnostics problems of constructive elements of different mechanical systems were investigated. In that paper the systems heterogeneity is related with the changes of its technical state or physical condition. Modelling of dynamics and analysis of correlation with system (object) state is the base for new diagnostic methods and algorithms.

The theoretical models of damaged continuous mechanical and heterogeneous multi-layered cylindrical systems with different features of elasticity were created. On this base new vibration diagnostic methods and algorithm for impulse impact optimization were made for directional system physical condition change. Applying analytical and finite elements models the dynamical characteristics were investigated and new vibration diagnostic methods and algorithm for systems defect level were suggested: it allows comparing the safety of monitoring objects and technological systems.

The diagnostic method based on Lamb waves' interference was created for mentioned systems, the methodology signal processing of Lamb waves' interference was based on two dimensional functions Fourier transformations and gradient of signal image characteristic parameters monitoring. It allowed determining the thickness of internal layer of heterogeneous cylindrical structure with significantly smaller outlays. The method which allows controlling interference of Lamb wave and achieves better quality of information of diagnostic signals in the cylindrical systems inner wall state control was developed too.

Introduction

Failure of mechanical systems (structures, mechanisms, machines) very often causes different accidents in industry. Therefore, problems of increasing operating reliability, diagnostics of technical state and condition monitoring of mechanical systems are always of great importance. Urgency of the latter first of all depends upon the complexity and danger they may cause to people and for the environment of the technical objects that include such mechanical systems. Operating reliability of different objects depends upon many factors and each of them have great influence upon a general level of the technical provision for normal functioning of the object and show the perfection of the object too. The latter includes such concepts as safety, efficiency, economic optimum, etc.

In that context the mechanical systems having high requirements of reliability and operating under a wide-range of different loads is the most critical point in avoiding industrial errors and achieving the perfection of technical objects. Systems heterogeneity is related with the changes in object technical state or physical condition.

One of the ways to increase the mentioned systems is to provide them with monitoring and diagnostic equipment based on the analysis of a vibroacoustical process. Modelling of dynamics and analysis of correlation with object state is the base for new diagnostic methods and algorithms. Practical application of this approach is restricted for mechanical systems by their complex dynamic behaviour under diverse loads as well as by the complicated influence of defects of the system as a whole. Therefore, for the implementation of diagnostics and monitoring it is to concentrate the important efforts on the solution of the following problems:

- identification methods of defects and their analysis in a qualitative relation to the location of the damage and its importance [1, 2];
- mathematical simulation of structure elements and of their dynamic behaviour at stiffness reducing damage [3, 6, 7];
- development of novel techniques for the state diagnosis of mechanical systems, based on the dynamic behaviour in the region of the damage to yield reliable signals of high information rates and determination of the specific features of mechanical systems of the given type [3, 4, 8, 9, 12].

Elaboration of techniques of diagnostics and common approach to this problem should be based on dynamic models and on the identification of mechanical systems containing the defects like reduction of stiffness, for instance. Some aspects of that approach and a number of diagnostics methods which have been developed for condition monitoring of mechanical systems are presented below.

Diagnostics of oblong constructive elements

The determination of defective structural elements is an important problem of diagnostics of various machines and mechanisms. When solving such a problem, for instance, by the method of resonance frequencies [1, 5, 7], a defective structural element is judged by the deviation of resonance frequency from the rated value.

Location of a defect in the structure is a more complicated task. For a certain class of structures, such as beam structures, a vibration test is used [9]. In this method, the beam structure is successively loaded with an additional mass and a harmonic force used for exciting vibrations in the loaded zones. The operating frequency is determined from the preliminary measured amplitude-frequency characteristic when the additional mass is located in the center of the structure. The operating frequency is taken to be equal to the frequency which corresponds to two-thirds of the maximum amplitude on the rising slope of the measured amplitude-frequency characteristic. The presence and location of a defect are indicated by the relationship between vibration amplitudes of the additional mass and its location.

The method of vibroacoustic control merely allows the determination of the presence and location of the defect; the size of the defect can be estimated only qualitatively.

However, the qualitative conclusions about size of the defect can not solve the diagnostics problems if, for example, the resource of the structure is monitored.

The weak point of the vibroacoustic control method is the unknown quantity of the additional mass, on which sensitivity strongly depends.

In order to investigate the possibility of the quantitative determination of the defect sizes and the calculation of control conditions assuring the maximum sensitivity of the method, a model of the defective structure “beam with fixed ends” was developed, whose diagram is shown in Fig.1.

Let us introduce the defectiveness characteristic of the structure which is convenient for practical application:

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{D}\Delta\left(\frac{\partial y}{\partial x}\right) \tag{1}$$

Here, y is the deviation of points of the beam structure from equilibrium position, x is the coordinate, D is the defectiveness characteristic of the structure,

$$\Delta\left(\frac{\partial y}{\partial x}\right) = \left.\frac{\partial y}{\partial x}\right|_{b+0} - \left.\frac{\partial y}{\partial x}\right|_{b-0}$$

is the change of the first derivative of the deviation by a coordinate in the defect zone, and b is the coordinate of the defect.

The defectiveness characteristic D is the integral quantitative characteristic of the defect, independent of the microstructure of the defect. The physical meaning of the characteristic D can be understood if we multiply both parts of Eq. (1) by EI, where E is Young’s modulus of the considered structure and I is the cross-sectional moment of inertia of the structure. Then we obtain:

$$M = \frac{EI}{D}\cdot\Delta\left(\frac{\partial y}{\partial x}\right) \tag{2}$$

where M is the flexural moment in the zone of the defect.

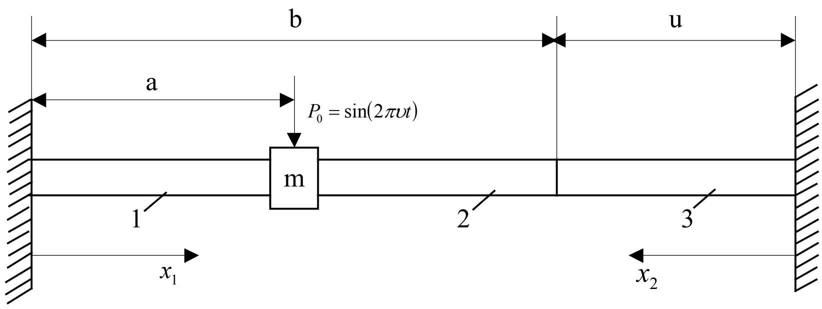


Fig. 1. The model of a defective beam with fixed ends: y_1 is the deviation of the *i*-th section of the beam structure of the equilibrium position; x_1 is the coordinate of the points of sections 1 and 2; x_2 is the coordinate of the points of section 3, *a* is the coordinate of the location of the additional mass *m*; *b* and *u* are the coordinates of the location of the defect; P_o is the amplitude of the existing force; ν is the excitation frequency; and *t* is time.

If $\lim\limits_{D\to\infty} M = 0$ - the structure is completely destroyed, and if $\lim\limits_{D\to 0} \Delta\left(\frac{\partial y}{\partial x}\right) = 0$ - the structure has no defect.

From Eq.(1) it is obvious that the defectiveness characteristic *D* determines the magnitude of the second derivative $\frac{\partial^2 y}{\partial x^2}$ at the fixed value of the change of the first derivative $\Delta\left(\frac{\partial y}{\partial x}\right)$ in the defect zone. For a smaller *D* the second derivative $\frac{\partial^2 y}{\partial x^2}$ in the defect zone will be greater.

Eq. (1) does not include parameters of the structure, such as *E, I, ρ, F, l* (*F* is the cross sectional area of the structure, *ρ* is the material density of the structure, and *l* is the structure length).

Only the characteristic *D*, the "action" in the form of $\Delta\left(\frac{\partial y}{\partial x}\right)$ and the "reaction" in the form of $\frac{\partial^2 y}{\partial x^2}$ are included. Therefore *D* characterizes the degree of destruction of the structure independently on the structure parameters (i.e., on *E, I, ρ, F, l*). The latter fact allows the comparison of the structures of different parameters with respect to the degree of destruction.

The flexural vibrations of the defective structure are described by the general differential equation:

$$\left.\begin{aligned} &EI\frac{\partial^4 y_1}{\partial x_1^4} + \rho F\frac{\partial^2 y_1}{\partial t^2} = 0; \qquad x_1 \in [0,a] \\ &EI\frac{\partial^4 y_2}{\partial x_1^4} + \rho F\frac{\partial^2 y_2}{\partial t^2} = 0; \qquad x_1 \in [0,b] \\ &EI\frac{\partial^4 y_3}{\partial x_2^4} + \rho F\frac{\partial^2 y_3}{\partial t^2} = 0; \qquad x_2 \in [0,u] \end{aligned}\right\} \tag{3}$$

Eq. (3) is solved by the method of separation of variables.

The mode of vibrations has the following form:

$$\left.\begin{aligned} Y_1(x_1) &= C_1^{(1)}S(kx_1) + C_2^{(1)}T(kx_1) + C_3^{(1)}U(kx_1) + C_4^{(1)}V(kx_1) \\ Y_2(x_1) &= C_1^{(2)}S(kx_1) + C_2^{(2)}T(kx_1) + C_3^{(2)}U(kx_1) + C_4^{(2)}V(kx_1) \\ Y_3(x_2) &= C_1^{(3)}S(kx_2) + C_2^{(3)}T(kx_2) + C_3^{(3)}U(kx_2) + C_4^{(3)}V(kx_2) \end{aligned}\right\} \tag{4}$$

Here $S(kx_i)$, $T(kx_i)$, $U(kx_i)$, and $V(kx_i)$, are Krylov's functions; $Y_i(x)$ is the function describing the mode of vibrations, ν is the frequency of vibration, and i is the index of the section of the defective beam structure;

$$k = \sqrt[4]{\frac{\rho F}{EI}(2\pi\upsilon)^2}$$

is the frequency parameter, and $C_k^{(l)}$ are constants which are found from boundary conditions of the considered defective beam system (see Fig.1).

According to Fourier method we obtain the system of linear equations relative to constants $C_k^{(l)}$ in the matrix form:

$$\begin{pmatrix} U(\alpha) & V(\alpha) & -S(\alpha) & -T(\alpha) & -U(\alpha) & -V(\alpha) & 0 & 0 \\ T(\alpha) & U(\alpha) & -V(\alpha) & -S(\alpha) & -T(\alpha) & -U(\alpha) & 0 & 0 \\ S(\alpha) & T(\alpha) & -U(\alpha) & -V(\alpha) & -S(\alpha) & -T(\alpha) & 0 & 0 \\ -AU(\alpha)-V(\alpha) & -AV(\alpha)-S(\alpha) & T(\alpha) & U(\alpha) & V(\alpha) & S(\alpha) & 0 & 0 \\ 0 & 0 & BU(\beta)+V(\beta) & BV(\beta)+S(\beta) & BS(\beta+T(\beta))+ & BT(\beta)+U(\beta) & T(\gamma) & U(\gamma) \\ 0 & 0 & U(\beta) & V(\beta) & S(\beta) & T(\beta) & -S(\gamma) & -T(\gamma) \\ 0 & 0 & S(\beta) & T(\beta) & U(\beta) & V(\beta) & -U(\gamma) & -V(\gamma) \\ 0 & 0 & T(\beta) & U(\beta) & V(\beta) & S(\beta) & V(\gamma) & S(\gamma) \end{pmatrix} \times \begin{pmatrix} C_3^{(1)} \\ C_4^{(1)} \\ C_1^{(2)} \\ C_2^{(2)} \\ C_3^{(2)} \\ C_4^{(2)} \\ C_3^{(3)} \\ C_4^{(3)} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ P \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad (5)$$

where

$$A = \frac{mP}{\rho F}k, \qquad B = kD, \qquad P = \frac{P_0}{Elk^3}, \qquad \alpha = ka, \qquad \beta = kb, \qquad \gamma = ku\,.$$

The constants $C_1^{(1)}$, $C_2^{(1)}$, $C_1^{(3)}$, $C_2^{(3)}$ are equal to zeros and do not enter into (7)

The dependence of the vibration amplitude $Y_1(y)$ of the additional mass m on the coordinate a, as it follows from (4), is expressed as

$$Y_1(a) = C_3^{(1)}U(ka) + C_4^{(1)}V(ka) \qquad (8)$$

Here, constants $C_3^{(1)}$, $C_4^{(1)}$ are found by solving the matrix equation (5) which can be compressed as

$$[K]\cdot\{C\} = \{F\} \qquad (9)$$

From the matrix equation, it is obvious that parameter P entering into the vector $\{F\}$ has no influence on the shape of the curve representing dependence of the vibration amplitude on an additional mass $Y_1(a)$. Parameter P can only increase or decrease uniformly the vibration amplitudes of the additional mass. Therefore, the amplitude of the existing force P_0 included in the parameter P, also has no influence on the shape of the curve $Y_1(a)$. The shape of this curve is completely determined by the matrix $[K]$.

The matrix $[K]$ also describes completely the defective condition of the beam structure.
If the following conditions are fulfilled

$$A = \frac{m}{\rho F} k = const \tag{10}$$

$$G = kl = const \tag{11}$$

where the letter G denotes the second constant, then the shape of the curve of the dependence of the vibration amplitude of the additional mass $Y_1(a)$ on the location of the additional mass a in the case of a defectless structure (D = 0) will be the same, regardless of the parameters of the considered structure (i.e., E, I, ρ, F, l). This fact allows a choice of the best control conditions with respect to the maximum sensitivity of the method. For this, it is sufficient to choose such values of the constants A and G which would provide the maximum sensitivity of the method and maintain them during the control procedure.

If the structure is defective ($D \neq 0$), then if the conditions (10) and (11) are satisfied, the shape of the curve $Y_1(a)$ would depend only on two parameters $B = kD$ and the relative coordinate of the defect location b/l. It can be seen from the expression of the matrix $[K]$. The relative change of the vibration amplitude of the additional mass in the defect zone would also depend on the same two parameters, i.e. we may write the functional relationship

$$\frac{Y_1(D, b/l) - Y_1(0, b/l)}{Y_1(0, b/l)} = f_1(B, b/l) \tag{12}$$

Here, on the left hand side the expression of the relative change of the vibration amplitude of the additional mass in the defect zone is represented. This expression can be denoted as $\Delta Y/Y$.

It is not difficult to determine the functional correlation between $\Delta Y/Y$ and B [9].

From conditions (10) and (11) the excitation frequency of vibrations υ and the magnitude of the additional mass can be expressed as

$$\upsilon = \frac{G^2}{2\pi} \sqrt{\frac{EI}{\rho F} \cdot \frac{1}{l^2}} \tag{13}$$

$$m = \frac{A}{G} \rho F l \tag{14}$$

The experiments have shown that for specific values of the constants, (13) and (14) can be rewritten as

$$\upsilon = \frac{0.84}{l^2} \sqrt{\frac{EI}{\rho F}} \tag{15}$$

$$m = 6.5 \rho F l \tag{16}$$

are the optimal conditions of the method of vibroacoustic control.

In Fig.2 the dotted line represents the characteristic shape of correlation between the vibration amplitude $Y_1(a/l)$ of the additional mass and its location a/l for the defectless structure when conditions (15) and (16) are satisfied. The solid line represents the dependence $Y_1(a/l)$ when the defect is present with the defectiveness characteristic D=1.27x10^{-2}m is present in the point b/l =0.7.

Having values of the constants A and G, we substitute the approximated relationships $H_i(b/l)$ into (12) and obtain an expression for the determination of the defectiveness characteristic D from the relative change of the amplitude of vibrations of the additional mass in the defect zone $\Delta Y/Y$, the coordinate of the defect b and the length of the structure l:

$$D = \frac{0.44 \cdot l \cdot \Delta Y/Y}{315 \exp\left(-\sqrt{115 |b/l - 0.5|}\right)(1 + \Delta Y/Y) - 1.6 \Delta Y/Y} \tag{17}$$

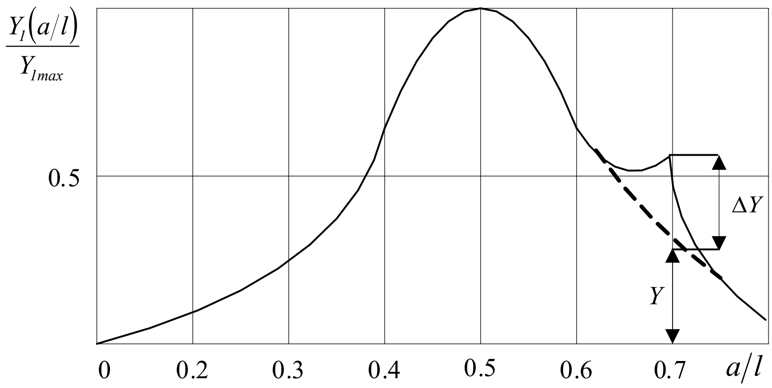


Fig. 2. Calculated correlation between vibration amplitude of the additional mass $\gamma_1(a/l)$ and its location *a/l*. Dotted line represents the defectless structure, solid line - when the defectiveness characteristic is $D = 1.27 \times 10^{-2}$m. ΔY is the change of the amplitude in the defect zone. Coordinate of the defect is *b/l* = 0.7, and *Y* is the vibration amplitude of the additional mass in the defectless structure when the coordinate of its location is *a/l* = 0.7

In conclusion it should be emphasized that for the determination of the defectiveness characteristic *D* it is sufficient to calculate the operating frequency by Eq.(15), magnitude of the additional mass by Eq.(16), to measure vibration amplitude of the additional mass vs. its location, to determine the relative change of the vibration amplitude of the additional mass in the defect zone *ΔY/Y*, and to calculate the defectiveness characteristic *D* by Eq.(17).

Diagnostics of axis - symmetrical constructive elements

Evaluation of technical condition of structures which is axis - symmetrical can be carried out using several known methods. Often their application requires vast and laborious experimental investigations in order to obtain sufficient information about maximally available meanings of parameters, by which quality of construction is characterized. Thus, we have a task of effective simulation and measurement of dynamical parameters of structures, which can be solved on the basis of numerical methods, i. e. of finite elements method (FEM). That gives to us the possibility to evaluate quality of structures by comparison of results of its dynamical simulation and experimental changes of resonance frequencies.

Let us consider the structure as multi-layered element consisting of enclosed one to another shells and cylinder axis - symmetrical beams. Mutual mounting of these parts is considered to be ideal, i. e. we ignore elastic plastic properties of inter-surface layers. Such an approach refers only to a slight difference between results of computation and experiments, if we keep in mind that no shearing deformations of inter-surface layers occur. Starting with practical considerations of frequentative method of vibrodiagnostics, we will examine bending modes of oscillations of structure. We can describe class of structures mentioned above more completely by two - dimensional composite finite elements operating on bending and extension - compression. Such an element is presented in Fig. 3. Located along axis *Ox*, it is bending on plane *xOz* and its cross section is treated either as radius in the case of the beam or as circular one in the case of multi-layered elements with thickness of individual layers $h_i = r_i - r_{i-1}$ (r_i, r_{i-1} - external and internal radii correspondingly).

In conformity with the theory of beams deformation at extension is considered to be constant within the whole area of cross section, and deformation at bend - proportional to the distance form neutral plane.

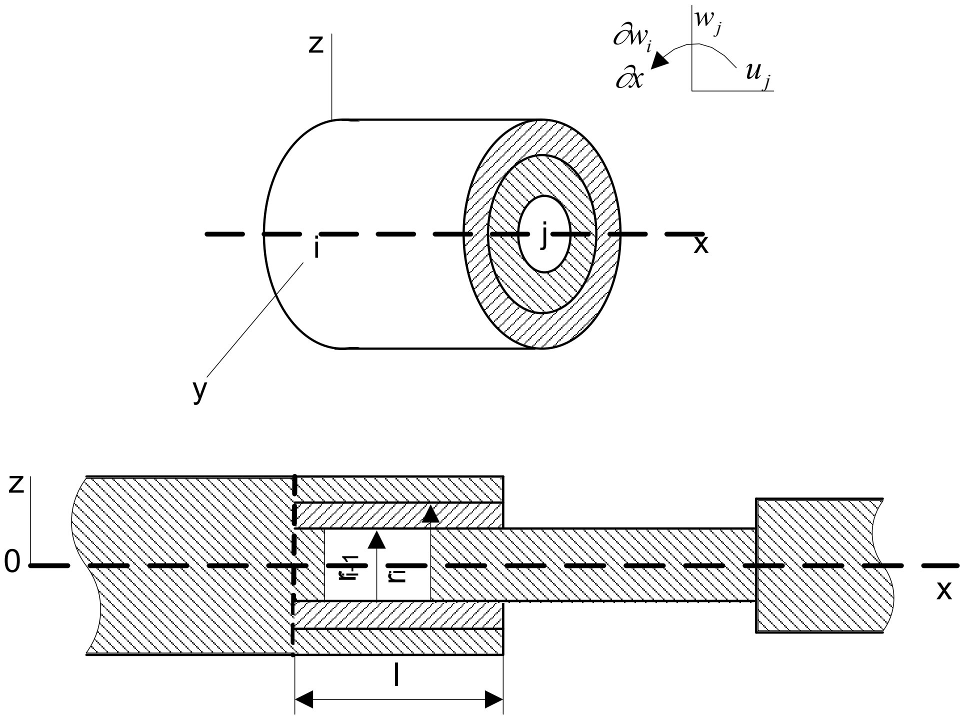


Fig. 3. Axis - symmetrical multi-layered element and its location inside of structure

The following vector is regarded as vector of node displacements of FEM:

$$U = \left\{ u_i \, , w_i \, , \, \frac{\partial w_i}{\partial x} \, , \, u_j \, , \, w_j \, , \, \frac{\partial w_j}{\partial x} \right\}^T \tag{18}$$

Using LaGrange equation and expressions for potential and kinetic energy we obtain system of equalities for dynamic balance of FE:

$$M_e \ddot{U}_e + K_e U_e = 0, \qquad K_e = \sum_{i=1}^{n} K_i \, , \quad M = \sum_{i=1}^{n} M_i \tag{19}$$

where

$$K_i = \begin{bmatrix} \frac{E_iF_i}{0} & 0 & 0 & -\frac{E_iF_i}{l} & 0 & 0 \\ & 12\frac{E_iI_i}{l^3} & 6\frac{E_iI_i}{l^2} & 0 & -12\frac{E_iI_i}{l^3} & 6\frac{E_iI_i}{l^2} \\ & & 4\frac{E_iI_i}{l} & 0 & -6\frac{E_iI_i}{l^2} & 2\frac{E_iI_i}{l} \\ & & & \frac{E_iI_i}{l} & 0 & 0 \\ & & & & 12\frac{E_iI_i}{l^3} & -6\frac{E_iI_i}{l^2} \\ & & & & & 4\frac{E_iI_i}{l} \end{bmatrix}$$

$$M_i = \rho_i F_i \begin{bmatrix} \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & 0 \\ & \frac{13}{35} & \frac{11}{210}l & 0 & \frac{9}{70} & -\frac{13}{420}l \\ & & \frac{1}{105}l^2 & 0 & \frac{13}{420}l & -\frac{1}{140}l^2 \\ & & & \frac{1}{3} & 0 & 0 \\ & & & & \frac{13}{35} & -\frac{11}{210}l \\ & & & & & \frac{1}{105}l^2 \end{bmatrix}$$

Here E_i, ρ_i, I_i, F_i - Young's modulus, material density, inert moment of cross section around the neutral axis and area of cross section correspondingly. Considering that section of *i*-th layer is circular one, we figure out:

$$I_i = \frac{\pi r_i}{4}\left(1 - \frac{r_i^4}{r_i}\right);$$

$$F_i = \pi\left(r_1^2 - r_{i-1}^2\right).$$

Natural frequencies and forms of oscillations of system can be determined by solution of a known equality

$$\left(K - \omega^2 M\right)U = 0 \qquad (20)$$

As a result we obtain *n* natural angular frequencies $\omega_1, \omega_2, ..., \omega_n$ with corresponding vectors $u_1, u_2, ..., u_n$ that determine forms of oscillations. Thus, it becomes possible to evaluate theoretically change of frequencies and forms of oscillations under concrete technical condition of structure. The latter depends upon the level of damage of structure (defect), specifically on quality of structure simulated by the change of stiffness of *j*-th element. In that case elements of matrices K_e and M_e are changing according to physical mechanical parameters of *i*-th layer or several layers. Radial defect is simulated with resolving power to one FE, longitudinal defect - by changing stiffness of several consecutive FE.

On the basis of the mentioned above algorithm computation of natural frequencies of multi-layered axis - symmetrical structures of connection was performed. Several structures were selected for computation, one of them is presented in Fig. 4. Analysis of natural frequencies of structure showed that frequencies for selected parameters of structural geometry depend upon stiffness of internal layers and geometry of typical sectors. This was observed, for example, when changing Young's modulus for internal beam (plastic) within limits $E = (0.03...0.07)\cdot 10^{11}\, n/m^2$ with density $\varrho = 1.4\cdot 10^3\, kg/m^3$ and change of section width 0…1.

The latter allows to diagnose the quality of connection of element's shell with plastic beam following the change of resonance frequency of oscillations (according to the results of simulation this change reaches 10%). The table presents several values of frequencies of structure depending upon stiffness of putting the shell on the plastic beam (model of defect oriented along the structure).

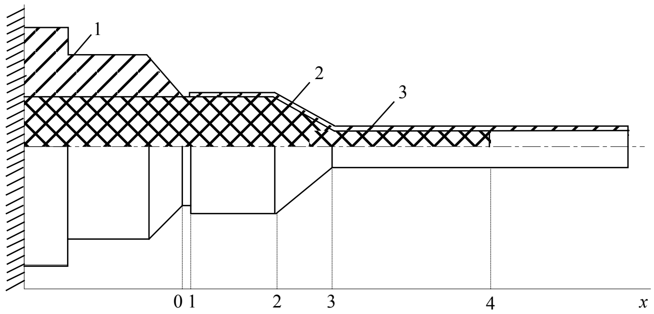


Fig.4. Multi-layered structure, consisting of the connection of axis - symmetrical elements: *1* - base, *2* - plastic beam, *3* - shell

Fig. 5 presents changes of natural frequencies for above mentioned structure with availability of longitudinal defect - low-quality connection of the shell and plastic beam. Difference between theoretical and experimental results is very slight, however it increases when changing boundaries of the qualitative connection.

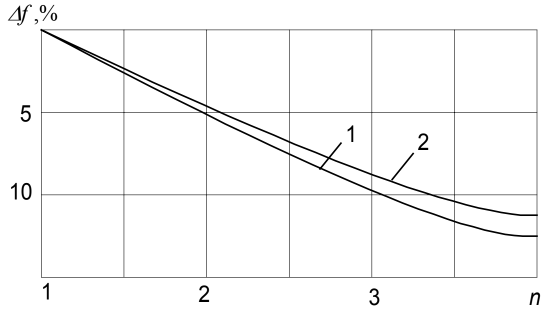


Fig. 5. Theoretical (*1*) and experimental (*2*) change of frequencies of structure depending upon boundary of qualitative connection of the shell and the beam

Now let us examine radial defect of structure. In the defect zone we have local stiffness *K*, and nobilities of the structural parts separated by defect are equal to *Q* and *Z*. Natural frequencies of such a structure are expressed

$$Q_x + Z_x + \frac{1}{K_x} = 0, \tag{21}$$

where index *x* - defines the coordinate along the axis *x*.

At qualitative connection we have $K_x \to \infty$, at low - quality connection (with defect) $K_x \to 0$, so quality can be expressed

$$\frac{1}{K_x} = -(Q_x + Z_x)_{\omega=\omega_n-\Delta\omega_n} \quad , \tag{22}$$

where ω_n - natural frequency on the *n*-th mode of oscillations, $\Delta\omega_n$ - its decrease owing to reduction of general stiffness of the complex mechanical system.

Let us examine concrete example of connection in point x_0 of two straight - lined beams with general length *l*. In case of harmonic impact on the mechanical system corresponding nobilities can be expressed

$$Q_{x_0} = -\frac{1}{EFp} ctgpx_0; \; Z_{x_0} = -\frac{1}{EFp} ctg[p(l - x_0)], \tag{23}$$

where $p = \omega\sqrt{\frac{\rho}{E}}$; ρ, E, F - density, section and Young's modulus for one - axis elements correspondingly.

Then, considering expression (21) for small $\Delta\omega_n$ when $\omega \neq 0$ and $0 < x_0 < l$, we obtain dependence

$$K_{x_0} = \frac{EF\pi n\sqrt{\frac{E}{\rho}}}{\Delta\omega_n l^2} \times \frac{1}{\cos ec^2\left(\frac{\pi n x_0}{l}\right)}, \tag{24}$$

which obviously defines current value of stiffness of connection K_{x_0} for every natural frequency ω_n and known parameters ρ, E, F. We should note that

$$K_{x_0 0} = \frac{EF\pi\sqrt{\frac{E}{\rho}}}{l^2}\frac{1}{\Delta\omega_1} = c\frac{1}{\Delta\omega_1} \tag{25}$$

when $x_0 = \frac{l}{2}$ and $n = 1$.

It means that decrease of stiffness at radial defect is inverse proportional to reduction of the value of first natural frequency of oscillations.

Obtained dependence allows make a monogram for quantitative evaluation of quality of connection for straight - lined beams and significantly simplify the procedure of vibrodiagnostics for beam structures. Besides, presented dependence, which characterizes damaging in point x_0 for one - axis element (when having simulation of defect by changing the local stiffness K_{x_0}) is an individual case of method for determination of defect location in machine structures on the basis of natural frequencies and forms of oscillations [7].

In that way it was demonstrated that with the help of the measurement of changes of resonance frequencies in structures with permanent connection and taking into consideration the preliminary simulation of extended and radial defects, we can evaluate the quality of the connection according to the results of the frequentative tests. Besides, for multi-layered axis - symmetrical structures we obtain qualitative tie of stiffness loss of connection and change of natural frequencies. For one - axis structures of the beam type analytical tie is obtained.

Diagnostics of pipelines based on the interference of Lamb's waves.

Internal layers of sediments (such as combustion char and like it) inside technological pipelines constitute one of the most important problems for chemical technologies of oil, power generation systems and for other pipelines. The sediments lower the flow, reduce the heat exchange and otherwise damage the performance of pipelines. A specific problem in measuring sediment layers is that the measuring value is integral in just short limited lengths or small areas, which considerably exceed the thickness of the pipe walls, while the measured scope along the pipe is very large.

The available techniques of measuring thicknesses of walls do not apply to layers of sediments, first of all because in order to find an integral value for a defined region, a large number of measurements should be performed at one point. A significant reflection from the internal surface, so important for either resonance or pulse measurement methods of thickness, is very difficult to find, because of the porosity of the sediments. A device has been developed recently for a similar application: it identifies corrosion - caused loss of thickness of the outside walls, by applying a two-dimensional Fourier transformation in the area of the wave number and of the frequency, when the axis of the transducer must be scanned. A modification of the construction is hardly possible because of the frequent mechanical and corrosive damages on the internal surfaces of the pipes, which are also often difficult to access.

Identification of waves propagating within the cylindrical structure. Among the variety of cylindrical structures there are the pipe-lines with the inequality $d<<R$, here: d - is the thickness of the wall, R - is an average radius of the wall.

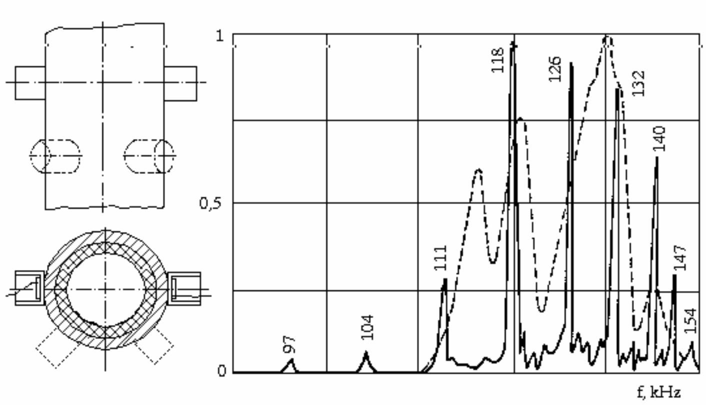
The segment of the wall slightly differs from the plate with thickness "*d*" where symmetrical and asymmetrical Lamb waves are easily excited:

$$U_{sz} = A \cdot F_s(m_s ,\ \omega, z, d) \cdot \sin (k_s x - \omega t) \tag{26}$$
$$U_{az} = A \cdot F_a(m_a ,\ w, z, d) \cdot \sin (k_a x - \omega t) \tag{27}$$

here: U_{sz} and U_{az} - displacements of the elements of the wall in the direction of the axis *z*, which is parallel with a normal line to the pipe surface; *y* axis is directed along the pipe; *A* and *B* - constants; F_s - even function (with respect to *z*); F_a - uneven function; m_s and m_a - order of the wave (0,1,2,...); ω - frequency; k_s and k_a - numbers of the wave.

It is characteristic of functions F_s and F_a that their values are not equal to "O" in the surfaces of the wall when $z = d/2$. The latter circumstance is very important in our case, when a layer of the sediment is formed and it firmly sticks to the wall. The waves excite displacements also in the direction of axis *x*, that will not be taken into account when considering the excitation and the reception of the waves we use.

It was attempted the diagnoses with waves of lengths by far exceeding both the thicknesses of the pipe-walls and of the sediments. The only waves capable of propagation in such conditions are the Lamb waves of different orders. Normally zero-order symmetric and asymmetric waves are easily excited. A longitudinal wave-transducer with either a flat or a convex surface was used to insure a proper contact at the necessary point for surfaces of arbitrary properties. Waves induced by such a transducer in the pipe-wall have been identified by an interferometer and by a pulse meter of the velocity and dispersion of the wave. They were the zero-order asymmetric Lamb waves, that propagate both along and normal to the pipe axis, and in all other possible directions, while their normal components interfere.



a) b)

Fig. 6. Locations of transducers (a) and interference signals (b). Solid line - a clean pipe, dotted line - a pipe with 1.5 to 2 cm sediment of coke. External and internal diameters of the steel pipes - 150 mm and 134 mm, respectively; *R*=142mm - an average diameter.

The figures in Fig.6 at the peaks of the interference stand for frequencies in kHz. Each of them corresponds to a wave number differing by one of length $2\pi R$ on the mid-line of the wall.

Algorithm for the estimation of the layer of sediments [11]. The layer of sediments is a damper, its thickness is determined by the quality of the interference system. But the resonance peaks are often distorted by the thickness of the pipe walls changing from corrosion or by a non-homogenous layer of the sediment. Therefore quality evaluations of the system, be it by the amplitude or by the width of the spectrum peaks, are fairly error-corrupted. During the compilation of the performance algorithm, a hypothesis was suggested. It says that signals of this type are a superposition of several interference peaks of similar frequencies. A set of situations was simulated on a computer, and the algorithm was selected as the one with the lowest response to the variations of the frequencies:

$$M = \frac{\int_1^2 \left|\frac{dU^{\copyright}}{df}\right| \cdot \frac{df}{dt} \cdot dt}{\int_1^2 U^{\copyright}\, dt} = \frac{\int_1^2 \left|\frac{dU^{\copyright}}{dt}\right| \cdot dt}{\int_1^2 U^{\copyright}\, dt}, \qquad (24)$$

here M - readings on the indicator scale of thickness units, $U'=U(1-k)$, U- voltage of the receiver, $k\cong 0.1$ - the value of truncating the effective noise. Ranges of integration 1 and 2 are relative indications of the frequencies which include the selected peaks. Their spacings are of the same order as the spacing between the two peaks.

One more problem with the samples of limited lengths is related to two overlapping systems of the interference: those of longwise and crosswise propagation of waves in the pipe. Therefore, special means are envisaged in the suggested algorithm [10]:

- eliminating one of the systems of the interference peaks;
- correcting the introduced errors.

The results of our study on the technique of the interference were implemented in an indicating device for the thickness of sediments, char in particular, inside technological pipes [11, 13]. The indicator is calibrated for pure pipes and pipes with admissible and inadmissible layers of sediments. The indicator (see Fig. 7) consists of an ultrasonic generator connected with the first transducer. This is connected acoustically via the pipe under control with the second transducer, the latter - electrically to an amplifier.

Fig. 7. General view of a portable indicator.

Output of the amplifier is connected to an integrator and a differentiator. Its output is connected via the detector to a divisor together with the output of the integrator, while the output of the divisor is connected to the indicating unit. This indicator is used for the diagnostics before the maintenance operation of pipelines.

Correlation between the condition of the pipe and characteristics of interference processes is another important question in investigation of the problem. It is possible for different thickness of the sediment in the pipe to carry out the measurement of the parameters of interference. But for that case we must to develop a method that would enable us to determine diagnostic parameters reliable enough. We apply autocorrelation function of frequency response, which is measured using transducer-receiver [14]. The autocorrelation function easy shows the differences – maximum steepness of that function in the internal units its first local trough is strictly connected with the attenuation of waves in the wall of resonance pipe.

Discussion of the results

Creation of the models of dynamics and evaluation of the technical state of prolong constructions by the identification of the heterogeneous mechanical systems with distributed parameters is effective and universal enough for the mathematical simulation of the dynamics of damaged constructions, for the analysis of the defect influence and for the search of informative diagnostic parameters. A number of methods for diagnostics of prolong constructions based on the analysis of the interaction of damaged constructions with added elements have been theoretically founded.

Lamb's waves can be applied for creation of new non-destructive measuring instruments of inside sediment layer in pipes. Propagation of Lamb's waves (flexural) in a pipe with or without an internal circular non-homogeneous layer was investigated. Two resonance systems are obtained: a system for waves that propagate perpendicular to the axis of a pipe, that is along the perimeter and another system for waves that propagate along a pipe. The investigation analyses the mechanism of asymmetric Lamb wave interference and the features of the phenomenon, dependent on the state of multi-layered cylindrical structure. It is shown that the state of cylindrical structure, which is characterized by sediment layer, formed during technological processes, can be controlled by analyzing amplitude frequency function of the structure. The method and the algorithm to analyse each resonance system and an indicating device are created. Experimental works are carried out: a new device is statistically examined and the basic ratio between the length and the diameter of a pipe is defined. Integral measurement, quick-action, especially simple mounting of transducers is typical to these measuring instruments.

Some monitoring systems including original diagnostic methods are developed and introduced in industry.

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