

# FEATURES OF THE ULTRASONIC WAVES REFLECTION FROM INHOMOGENEOUS BOUNDARY OF CONTACTING SOLIDS

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There are many articles used in industry and belonging to the layered materials and it is actually to inspect the quality of the layers's interface boundaries (*IB*) or joints, where, partially, one of the layers functions as a component of friction surface cohesion, as a protective coating, “contact power” (and etc.) of joints of two materials [1-4]. In spite of amplitude, spectral, phase and etc. methods of ultrasonic inspection, applied to evaluate quality of *IB*, the problems are to be arise when contacting materials have small surface of defect  $S_D$ , substantially different elastic properties, non-constancy of its structure and ultrasonic attenuation, high roughness of external surface and there is the only one-side access to inspected object. To overcome previous difficulties we are developing the method of the joints quality evaluation, firstly suggested in [5] and named as method of “Apertures and Phase Optimization of Imaginary Sources” (*APOIS*), where “imaginary sources” are the interface surfaces which coherently reflect incident ultrasonic waves (*UW*) of different amplitudes  $P_i$  and “shifted” phases  $\Delta\phi_i$ . This work is devoted to *APOIS* further development including application of volume, surface, subsurface and another modes, while classical “discrete” and continues boundary conditions exist on the *IB* surface.

**Analysis of the problem.** The first part of this work is devoted, mainly, to an acoustical path analysis of the *APOIS* method when one or the pair of angle probes are used to inspect the *IB* surface of contacting materials at which “free-slip”, “slip-rigid”, “rigid-free” and “continuously” varying inhomogeneous conditions are modelling (Fig.1).

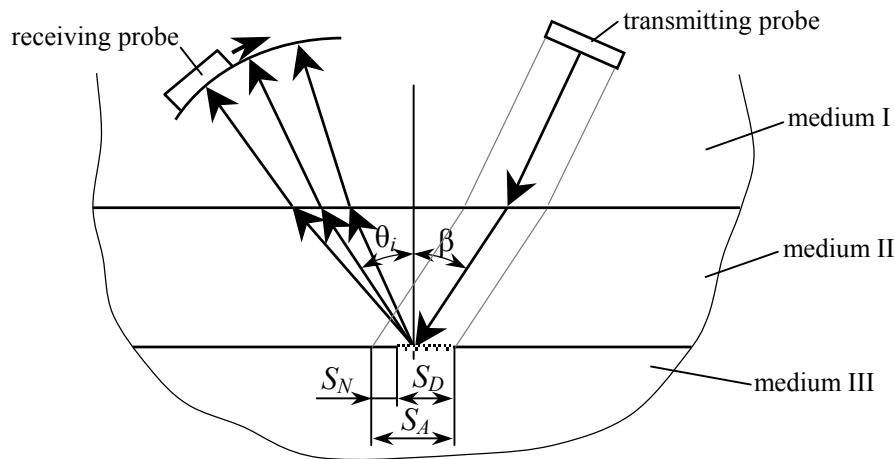


Fig.1. Acoustical path of APOIS method

At the beginning, calculation has been made of the mode's reflection coefficient  $\mathcal{R} = |R_{\ell\ell}, R_{tt}, R_{t\ell}, R_{\ell t}|$  and wave phase  $\phi$  according to classical formulas [6] at different ratio of ultrasonic velocities of contacting materials  $n = C_3/C_2$ , densities  $m = \rho_3/\rho_2$ , where index  $l$  is proper to longitudinal and  $t$  – transverse mode. And then conditions are defined, under which phase shift  $\Delta\phi$  of *UW* reflected from different boundaries is optimal. Partially, amplitude ( $R_{ll}$ ) and phase parameters ( $\Delta\phi$ ) of longitudinal waves, reflected from homogeneous *IB* surface ( $n > 1, m > 1$ ) vs. of angle of incidence  $\beta$  are in Fig.2.

As it follows from analysis of the former dependencies  $\mathcal{R}\{n,m,\beta\}$  there are conditions, including angle of UW incidence and reflection, modes using, direction of the object sounding, when the phase shift  $\Delta\phi$  of UW reflected from free, slip and rigid boundary can be from zero to  $\pm\pi$ . And this fact is important for realization of *APOIS* method.

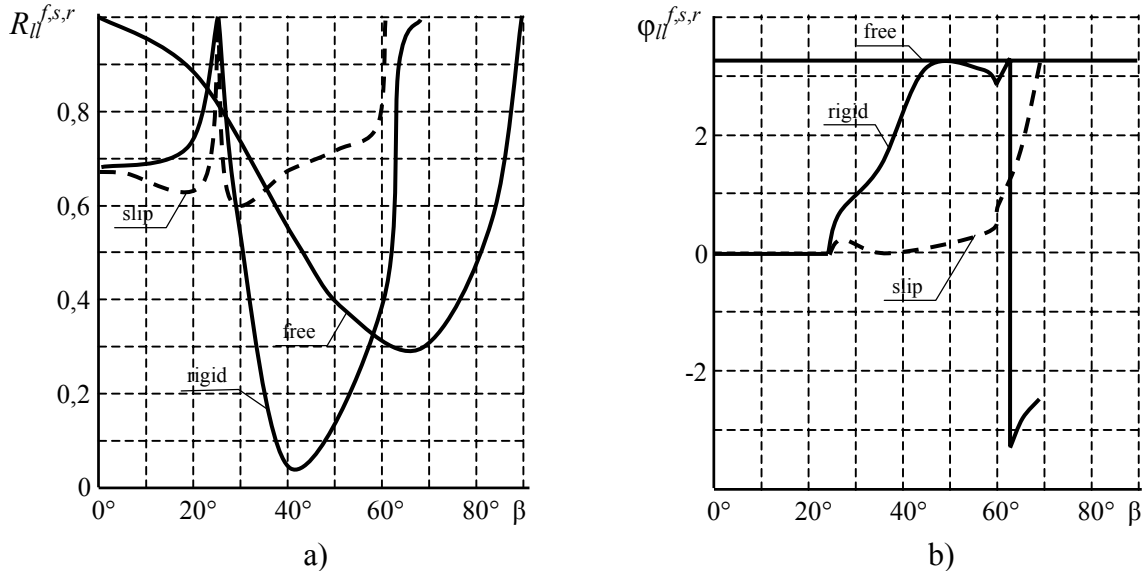


Fig.2. Example of amplitude (a) and phase (b) characteristics of wave reflected from boundary Plexiglas – Aluminum vs. angle of UW incidence

Let us consider (qualitatively) acoustical path of ultrasonic defectoscope when a pair of angle probes, used to inspect the quality of *IB* surface (Fig.1) by echo-method, is moving along the scanning surface in direction  $x$ . For this case amplitude of ultrasonic signal propagating from transmitting probe to receiving probe at point  $X_0$  is

$$P(X_0) \sim F_{D_{0,1}} F_{D_{1,0}} F_R, \quad (1)$$

where  $F_{D_{0,1}}$  is the integral function, characterizing parameters of *UW* propagating from transmitting probe through the boundary  $S_c$  up to *IB* surface;  $F_{D_{1,0}}$  – is integral function, characterizing acoustical parameters of *UW* propagating through the boundary surface  $S_c$  up to receiving probe;

$$F_R = \iint_{S_0} \Omega(\vec{r}_0 - \vec{r}_X, \delta, \lambda_1, \omega, \tau, \theta) R(X) \exp[i(\Delta\phi + \Psi)] dS = (F_R)_{Nf^+} (F_R)_D \quad (2)$$

is an integral function in the material layer 2, characterizing acoustical field of *UW* simultaneously reflected from *IB* surface with defect ( $S_D$ ) and non-defect ( $S_N$ ) surfaces and depending on the phase shift  $\Delta\phi(X)$  and amplitude of reflected waves at *IB* -  $P(X)$ ;  $X=X(x,y,z)$  - is coordinate of the local interface surface  $dS$ , reflecting *UW*;  $\vec{r}_0$  - radius vector of the observation point and  $\vec{r}_X$  - radius vector of the  $X$  coordinate;  $\theta$  - is an angle of the beam reflected and propagating in the material 2;  $\tau$  and  $\omega$  - are time duration and frequency of the *UW* signal pulse;  $\Psi$  - is wave phase of *UW* propagating from transmitting probe to *IB* surface. It is clearly, that functions  $\{F_{D_{0,1}}, F_{D_{1,0}}\}$  depend substantially on magnitude and variations (along  $x$ ) of the scanning surface roughness, material structure and *US* attenuation. So, to increase inspection reliability it is necessary to create such conditions at which function  $F_R$  (or amplitude and directivity of imaginary sources) undergoes substantial variation when acoustical beam, reflecting from *IB* surface of joints with defect (adhesion, cohesion, “contact power” etc.), is moving along  $x$ .

**Inhomogeneous Discrete Conditions.** For the sake of simplicity  $F_{D_{0,1}}$  and  $F_{D_{1,0}}$  are constants and the problem is assumed to be two-dimensional, and phases of the waves reflected from the interface surfaces  $S_N$  and  $S_D$ , which cross-sectional size not higher  $d$ , are not the same and difference between

them is  $\Delta\phi=\text{constant}$ . Then, formula for amplitude of the waves reflected from *IB* surface while angle  $\theta=\beta$  (Fig.1) and received by probe can be written

$$P \sim K_D S_D \cos\beta [1 + \exp(i\phi) \frac{K_N}{K_D} (\frac{S_A}{S_D \cos\beta} - 1)], \quad (3)$$

where  $K_D$  and  $K_N$  – are the integral coefficients to characterize *UW* path from transmitting to receiving probe, including wave propagation through the scanning surface (twice) and reflection from *IB* surface and depending on angular parameters, sizes of imaginary sources ( $S_N$  and  $S_D$ ), acoustical properties of contacting materials, *UW* frequency and time of pulse duration;  $S_A = S_D + S_N$  – is surface of an acoustical beam spot on the *IB*;  $S_{DN} = S_D/S_N = d_{DN} = d_D/d_N$  – may be  $>1$  or  $<1$ ;  $d_D$  is width of defect *IB* surface and  $d_N$  – of non-defect surface;  $d_{DA} = d_D/(d_N + d_D)$ .

From (3) follows that  $P \rightarrow 0$  (or  $|\log P/P_0| \rightarrow \infty$ ) if

$$d_{DN} = [1 - K_{DN} \exp(-i\Delta\phi) - 1] \cos\beta - 1. \quad (4)$$

That means that high sensitivity of the method to defects like “lamination” is achieved as a result of interference of the fields of neighboring “imaginary coherent sources” of the waves, rather than due to a varying reflection coefficient of the obliquely incident wave during phase transformations at homogeneous boundary [6]. As it follows from the calculation data, we can observe substantial variations of directivity  $\Phi(\theta')$  in the layer of material 2 – Fig. 3 (or  $\Phi$  vs.  $\alpha'$  in the medium 1 – after refraction at the boundary 2 $\rightarrow$ 1) vs. position of the incident beam  $x$  with regard to the boundary line  $L_{DN}$  between reflecting spots with different boundary conditions, where  $\theta' = \theta - \beta$ .

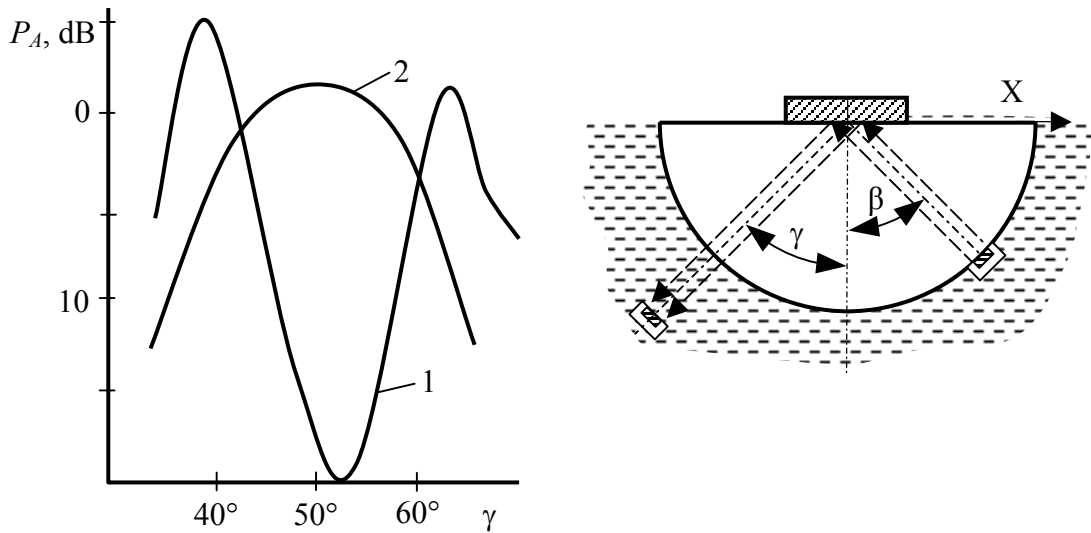


Fig.3. Acoustical field of UW reflected from *IB* surface slip-free (1) and free one (2): contacting materials Plexiglas – Steel;  $\beta = 50^\circ$ ; position of  $L_{DN}$  line  $X=0$

At that one can observe the shift of angular maximum  $P(\theta)$  or/and appearance of two or more additional maximums, etc. (Fig.3. 4 ) (I.e. it is assumed that when inspecting the quality of surface cohesion, one can realize such conditions, at which registered acoustic field variations of the reflected waves from defective surfaces were maximal and measurement sensitivity was the highest). Let  $\beta = \theta$ , then the dependence of  $P$  vs. position of the transmitting probe  $x$  (or incident beam) has extremes independently on  $\Delta\phi$  sign. And function  $P(x)$  has the only minimum at some  $d_{DN}$ , when the straight line  $L_{DN}$ , separating self infinitive defect and self non-defect surfaces, is normal to  $x$  and to the plane of incidence (Fig.5). It is confirmed by calculation data obtained when the probes are scanning in direction  $x$ . In this case we can observe *IS* directivity varying vs. probes position  $x$ , including appearance of two principal and/or additive lobes (at definite  $x^*$ ), which axes “rotate” in plane of incidence.

The dependencies like that can be got when the line  $L_{DN}$  is parallel to incidence plane and the probes are scanning in  $y$  direction. In this case the part of an acoustical energy flow of reflected waves is directed such way that the lobes to be appear have axes which are not in coincidence with

the incidence plane. If there is angle  $\gamma$  ( $\leq \pi/2$ ) between straight line  $L_{DN}$  and direction of the probes scanning  $x$  the divergence of the acoustical energy out of incidence plane grows vs.  $\gamma$ .

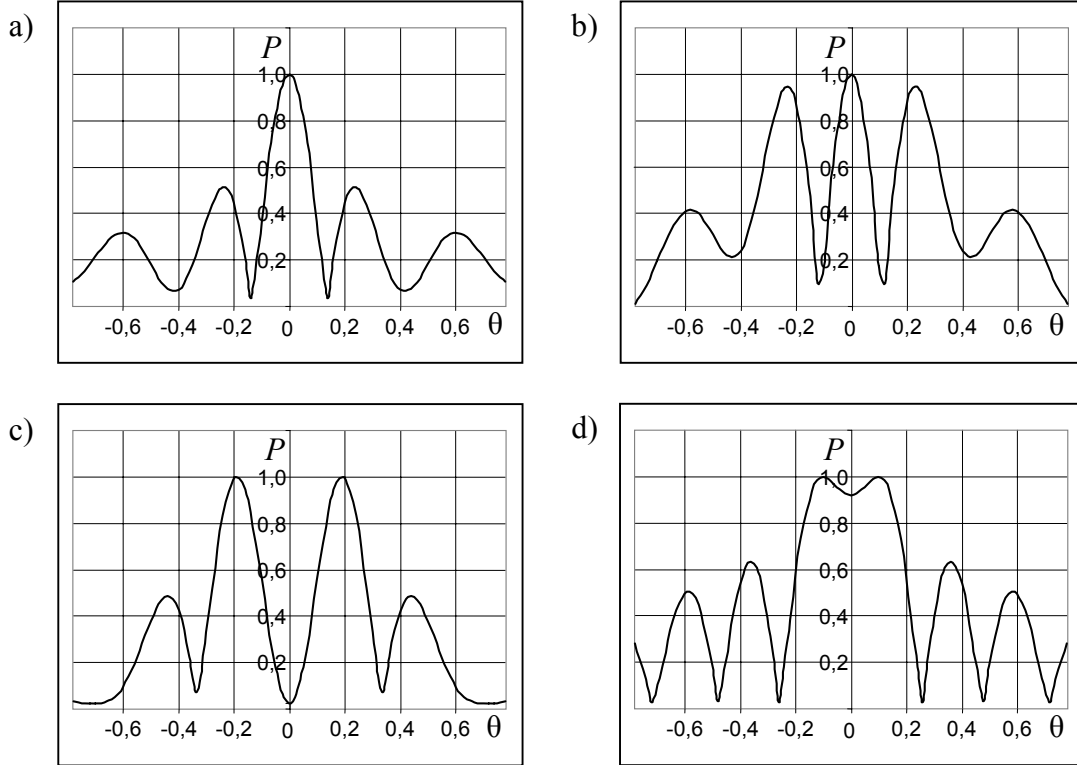


Fig.4. Evolution of the IS directivity:  $d_D/d_A=0,1$  (a);  $0,2$  (b);  $0,5$  (c);  $0,75$  (d)

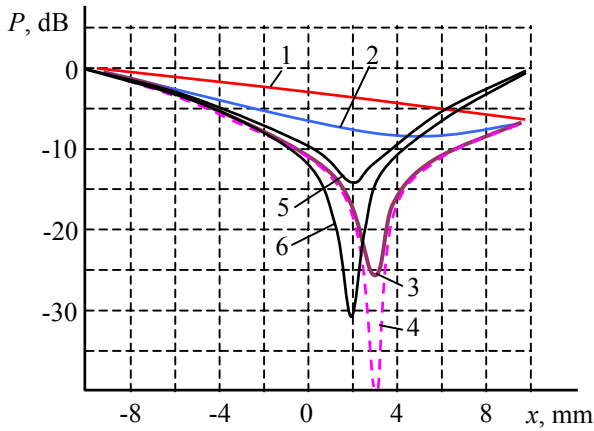


Fig.53. Characteristic dependencies of  $P$  vs.  $L_{DN}$  position  $x$  (1-5) and  $y$  (6) at different phase shift; materials: Plexiglas – Plexiglas; IB surface: free-slip; 1-4 – theory; 5,6 – experiment;  $\phi$ , rad = 0 (1); 2 (2); 3 (3);  $\pi$  (4)

sensitivity and reliability. It is confirmed by dependences of  $P(x)$  in Fig.6 where the transmitting and receiving probes are directed in the same direction and have incidence angle close to some  $\beta$  which is small. As it follows from calculated data the form of  $P(x)$  curve substantially depends on  $\beta$  and  $d_A l \lambda$  and may have simultaneously very deep minimum and sharp maximum for large  $d_A l \lambda$ . As it follows from the theoretical analysis of different ways of the APOIS realization we can conclude that the more varying of amplitude parameters  $P$  or the most sensitivity of the IB flaw detection can be achieved when the phase shift  $\Delta\phi$  between waves, reflected simultaneously from the former different boundaries ( $S_N$ ,  $S_D$ ) is nearly  $\pi$ .

The data obtained in Fig. 4 illustrate  $\Phi(\theta)$  evaluation vs. ratio of defect width  $d_D$  to an acoustical spot's width  $d_A$  when defect region is in the middle of an acoustical spot ( $d_{DA} \geq 1$ ). If  $d_{DA}=0$  there is the only principle maximum of directivity  $\Phi(\theta)$ . Then,  $d_{DA}$  increasing causes appearance of three principle maximums of the same amplitude at  $d_{DA} \approx 0,1$  and at  $d_{DA} \approx 0,2$  they are the have the same amplitude. With further of  $d_{DA}$  increasing a number of principle directivity maximums is reducing to 2 ( $d_{DA} \approx 0,5$ ) and at  $d_{DA}=1$  – maximum is alone and the  $\Phi(\theta)$  is the same as at  $d_{DA}=0$ .

There is the optimal receiving probe position and optimal angles  $\theta_i$  ( $\theta_i < \beta = \theta$ ,  $\theta_i > \beta$ ) at which the behavior of  $P(x)$  will be in another way, and there is possibility to increase the methodic's

**Inhomogeneous Continuous Conditions.** Really, boundary conditions can be another than the mentioned before. But if the optimal parameters of the acoustical path  $\{\beta, f, \tau, \Delta\phi, \theta, d_A\}$  and the probes aperture being determined there are conditions at which  $IS$  directivity varying and sensitivity measurements may be meaningful.

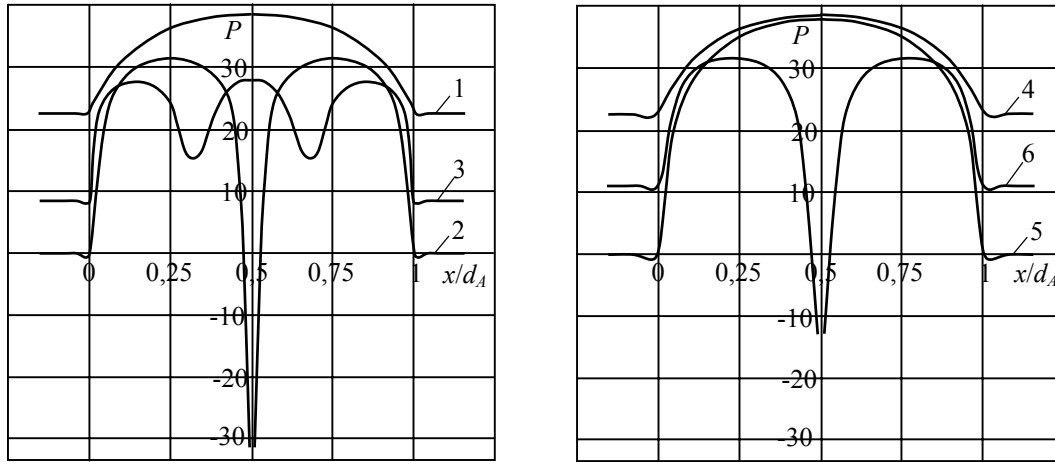


Fig.6. Dependencies of  $P(x/d_A)$  at different  $\beta$ , when receiving and incidence angles are nearly the same:  $kd_A = 40$  (1-3);  $20$  (4-6);  $\beta = 4^\circ$  (1);  $9^\circ$  (2);  $13^\circ$  (3);  $8^\circ$  (4);  $9^\circ$  (5);  $17^\circ$  (6)

For the sake of simplicity, the phase shift  $\Delta\phi(d_t)$  is linear function and occupies the central part of the acoustical spot -  $d_t \leq d_A$ . If  $d_t = 0$ , directivity of reflected waves  $\Phi(\theta')$  – is symmetrical function with two equal lobes which amplitudes  $P_1(\theta' < 0) = P_2(\theta' > 0)$ . As seen in Fig.7, the difference  $P_1 - P_2$  arise and increasing with increasing of the transition zone  $d_t$  up to  $d_t = d_A$ . At  $d_t = d_A$   $P_1$  and angle of  $|\theta|$  are maximal. As calculations show, the lesser the phase shift the lesser the angular and amplitude parameters of acoustical field of reflected waves.

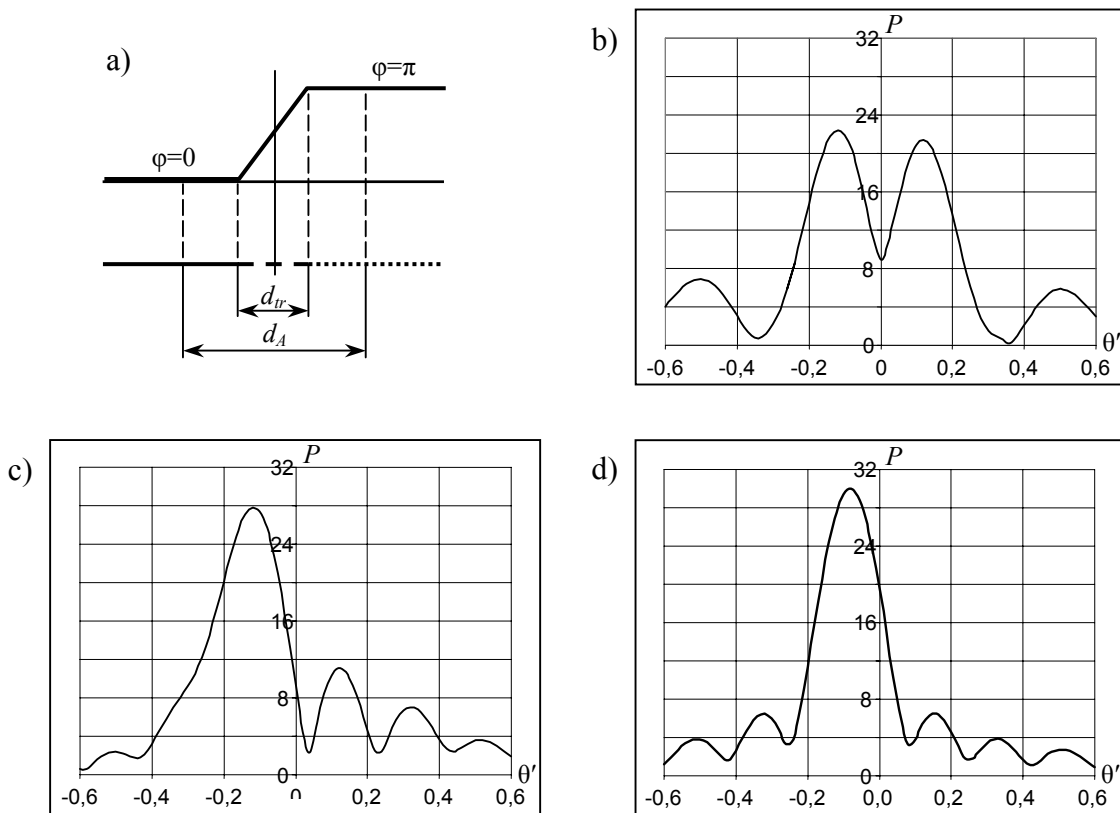


Fig.7. Evolution of diagram directivity of reflected waves at different phase transition zone:  $d_{tr}/d_A = 0,03$  (b);  $0,5$  (c);  $1$  (d)

**Experimental methodic.** To verify theoretical analysis, a setup and measurement technique, shown in Fig.3,8-11 have been developed. It was necessary to study of the possibilities of the longitudinal, surface, head, transverse waves using in *APOIS* method, and to reveal the  $\Phi(\theta)$  evolution when the boundary line, separating surface regions with different *IB* conditions, is translating or rotating. The nominal frequency used was 1-5 MHz and 10 MHz. Simulating of slipping boundary condition is realized by developing contact of plane-parallel surfaces of the materials through a thin liquid interlayer with width  $h$  and set the latter according to relation  $h^* < h < h^{**} < \lambda$ . Width  $h^{**}$  is boundary thickness of contacting interlayer, ensuring equality of strain normal components and absolute slipping of tangential component of incident wave shift. Rigid boundary is being simulated by sticking the materials and free one – by absence of their contact. Contacting materials are: Plexiglass-Steel, Plexiglass-Aluminum, Plexiglass-Plexiglass, as well as plexiglass-rubber, where load specimen is  $(30 \times 40 \times 10) \cdot 10^{-9} \text{ m}^3$ .

When we study the features of the volume waves reflection, *UW* via Plexiglass fall onto the media joint with simulated inhomogeneous boundary conditions, are reflected and received by receiving probe, which is provided with the opportunity of moving along plane, cylindrical or spherical surface of specimen. One can easily study the fields of the waves reflected from rigid-slip and rigid-free boundaries. Simulation of the boundaries with phase shift  $\pi$  for the reflected surface waves was realised by using Al-specimens, one of which have the form of rectangular parallelepiped and another one – was like that but with the projection. In the first case the boundary of surface waves reflection was the line of the specimen's boundary intersection. In the second case - the boundary of the waves reflection was the intersection line of the specimen's contact plane and internal plane of projection. For study subsurface longitudinal waves reflection from the free-slip boundary we used Plexiglas specimen which vertical boundary plane surface is contacting with the plane-parallel surface of the Steel specimen-reflector through a thin liquid interlayer ( $f=1 \text{ MHz}$ ); specimen-reflector is moving in vertical ( $z$ ) and in horizontal ( $x$ ) direction (Fig.8), by varying the  $L_{DN}$  position in space. The subsurface wave probes with local immersion bath (volumes of magnetic fluids held by magnetic system) working in duet-regime) have been used [7,8]. Experimental data has been obtained on the setup, which measuring circuit was assembled on the basis of standard devices. Respective units of ultrasonic flaw detector are the source and amplifier of the probing signal. The signal is given from amplifier 2 outlet to one of the screen sweeps of double-beam oscilloscope C1-71 to which a reference signal from test oscillator is sent at the same time to define probing signal amplitude by comparison method. Simultaneously amplitude stability as well as pulse shape in time are controlled by sending an electrical pulse from flaw detector's oscillator outlet to the second channel of oscilloscope sweep (via a divider). Circuit operation is synchronized by a device И2-26, which makes probing pulse delay and scanning, as well as measures time intervals.

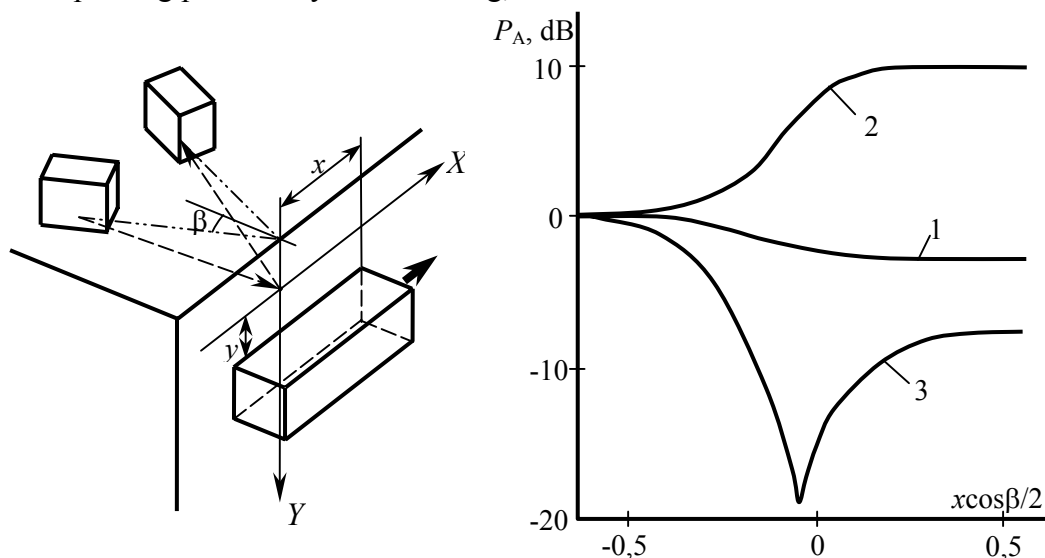


Fig.8. Experimental scheme and amplitude of head waves reflected from IB surface: free – slip vs.  $x$  and  $y$  shift of the  $L_{DN}$  line:  $y/d_A = 0,5$  (1); 0 (2);  $-1,2$  (3)

## Results of research and discussion.

Major results of experimental studies are given in Fig.5, 8-11. As follows from the data of the theoretical analysis and the laboratory data good qualitative agreement is observed between them. It should be noted that the parameters of reflected beam acoustic field considerably vary as far as line  $L_{DN}$ , separating regions of different boundary conditions relatively “moves” along axis  $x$  (straight line  $L_{DN} \parallel y$  and  $z=0$ ) is experimentally confirmed (Fig.5, 9). If  $\beta=\theta$  the behaviour of the signal amplitude dependence at the receiving probe on  $L_{ND}$  coordinate  $P(x)$  considerably varies, depending on the locality of media joint surface zones with different boundary conditions. And one can also observe separation (or “splitting”) of the main lobe of acoustic field into two with peak amplitude and reception angles  $\theta_1$  and  $\theta_1 < \theta = \beta < \theta_2$  depending on  $L_{DN}$  position and reflection factor difference in value (and phase shift  $\Delta\phi$  between them). The more is  $\Delta\phi$  the larger is angular shift of  $\Delta\theta_i = |\theta - \theta_i|$ , as well as that of coordinate  $x^*$  of  $L_{ND}$  line relatively  $x=0$ , at which minimal signal is observed at probe (Fig. 5). As laboratory data show, curve  $P(x)$  has one deep minimum and two additive small maximums ( $\sim 1-3\text{dB}$ ) at some  $x$  - nearly edges of an acoustical spot when free-rigid and free-slip boundary conditions are. But if the  $IB$  conditions are rigid-slip the former local maximums - absent. It is clearly that if receiving probe set at angle  $\theta_i$  dependence of  $P(x)$  may be increasing vs.  $x$  at some interval  $\Delta x$ .

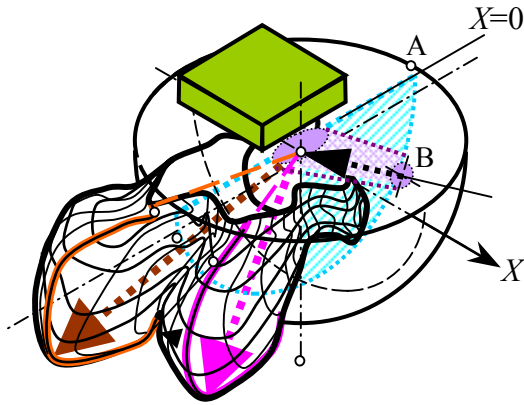


Fig.9. Scheme of experiment and field of waves reflected from the boundary free-slip:  $\beta=50^\circ$ ,  $x=0$ ;  $L_{DN} \parallel x$ ; materials: Aluminum – Steel

It is ascertained that acoustic beam splitting in to two and more is also observed during sequential motion of the limited-width acoustic beam in parallel to  $L_{ND}$  line, however at that, the location of the main plane of longitudinal wave reflection is changed too. Fig. 5 and 9 illustrate experimental data obtained by using two setup and are in a good qualitative accordance with analysis. It was obtained in practice that if the difference between reflection coefficients is not large and  $L_{ND}$ -line changes its angular direction relatively coordinates  $\{x, y\} \sim 0$  (or rotate) there to be appear two principle lobes which have maximum angular divergence when  $L_{ND} \perp y$ . To add, depending on the angle of incidence and reception of the acoustic signal, type of

boundary conditions, signal amplitude at the receiver can decrease by 20 – 40 dB.

**Surface waves.** There are verified the results of the analysis of the efficiency of the APOIS application in the case of surface waves using to inspect inhomogeneous boundary. Fig.10 illustrates two variants of APOIS application: a) receiving and transmitting probes placed on the contact surface symmetrically at angles  $\beta=\theta=45^\circ$  to the normal vector restored to straight line  $L_B$ .  $L_B$  is internal line created by straight corner of the specimen with rejection ( $L_{B1}$ ) and straight corner of parallelepiped  $L_{B2}$ , where  $L_B = L_{B2} + L_{B1}$ ; b) transmitting and receiving probes have nearly identical and small angle of the  $UW$  incidence and receiving. As there show (Fig.10), application of (a) APOIS variant of measurements (according to classical scheme) is not effective independently on incidence (receiving) angles varying from zero to  $45^\circ$  and more. It is explained by the fact that there is very high difference between reflection coefficients  $R_I$  and  $R_{II}$  at line boundary I ( $L_{B1}$ ) and II ( $L_{B2}$ ) where  $20\log(R_{II}/R_I)$  achieves by  $\sim 16\text{ dB}$  if  $\beta=\theta=0$ , but decreasing up to  $\sim 10\text{ dB}$  at  $\beta=\theta=45^\circ$ . (It is interesting, that the boundary II proper to artificial defect - infinitive crack). As there are show using of the methodic variant II we achieve substantial increasing of amplitude difference  $\Delta P$  between  $UW$  signal reflected from  $L_B$  with artificial defect and without it. The higher is the wave frequency the more is the  $\Delta P$  effect or the inspection sensitivity.

**Dual probe of longitudinal waves.** The scheme of research and the results of measuring are in Fig. 11. The dual probe is applied to scan Plexiglas-Steel specimen with artificial defects of long rectangular form and different width and free-rigid boundary is simulated.

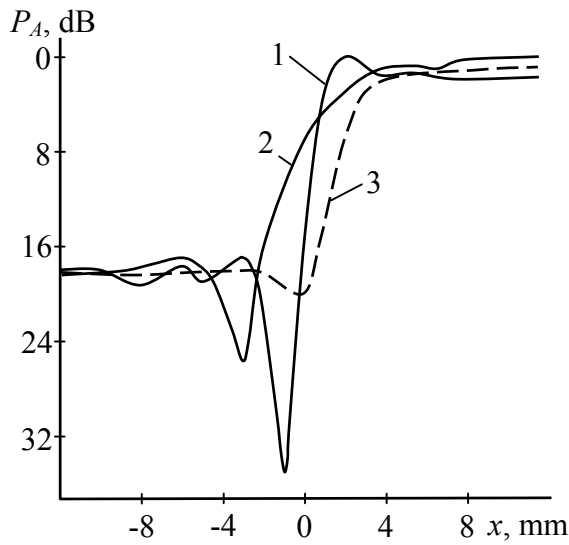


Fig.10. Experimental amplitude of Rayleigh wave, reflected from inhomogeneous boundary, vs. probe position  $x$ : frequency  $f$ , MHz = 5 (1); 1,8 (2, 3); variant I (1, 2); II (3)

The angles of probe prism are  $4^\circ$  and  $6^\circ$ . Behaviour of curves  $P(x)$  is in qualitative accordance with *APOIS* analysis and calculation data partially presented in Fig.4. It follows from laboratory data that the higher wave frequency and the sharper the probe's directivity the more the  $P(x)$  varying and the more the inspection sensitivity. Formulas 1 and 2 may be used to evaluate the sizes of small flaws when by using *APOIS* method applied. For this aim it is necessary to obtain continuously varying (increasing or decreasing) dependence  $P$  vs.  $d_{DA}=d_D/d_A < 1$  at some  $d_{DA} = 0-d_{DA}^*$ . But it is necessary take into account the effect of the phase shift  $\Delta\phi$  and different coefficients of *UW* reflection from *IB* surface.

**Subsurface longitudinal waves.** Since the main point of non-destructive method under consideration has wave nature, one should assume

similarity of dependencies considered for the case of using other wave modes too, including subsurface waves [7,8], transverse, plate waves and etc. Fig. 8 shows the effect of head longitudinal wave reflection from slip-free boundary, when  $L_{DN}$  line moves in two directions perpendicular to each other. As seen, even reflected signal multiplication is observed during specimen-reflector's motion along the normal direction towards contacting surface, which is caused by the shift of imaginary source emission field maximum in vertical plane due to interference phenomenon.

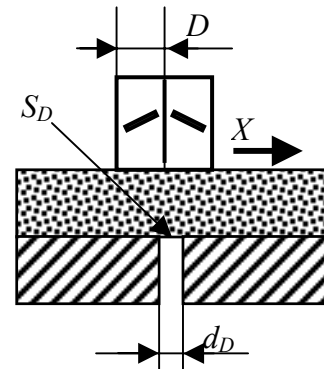
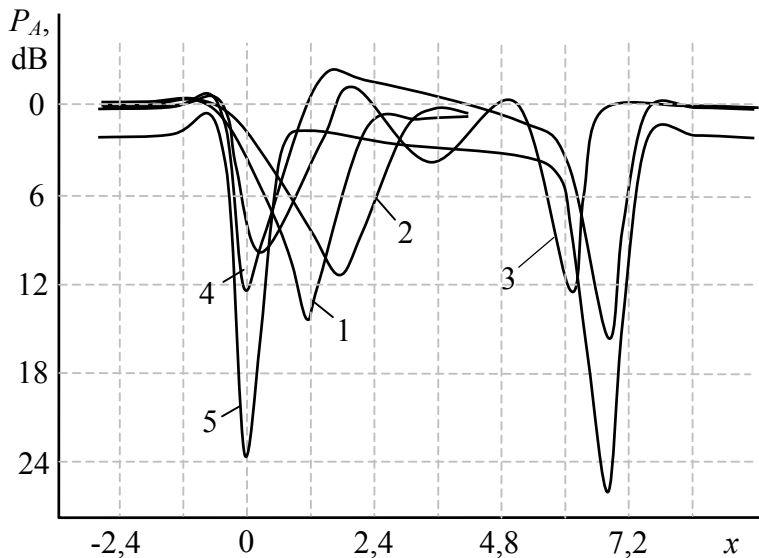


Fig.11. Amplitude of the longitudinal wave, reflected from *IB* surface (free – rigid), vs. dual probe position  $x$ :  $d_D/D = 0,36$  (1);  $0,54$  (2);  $1,44$  (3);  $1,8$  (4);  $2,9$  (5);  $f$ , MHz = 2,5 (1-4); 10 (5)

So, it is possible to conclude that the obtained data can be used for development of the *APOIS* inspection method and be useful for understanding of the results of two-layered materials inspection realized by the traditional way. This method is perspective to inspect the materials adhesion quality - to detect discontinuities of *IS* of contacting materials with identical or strongly different acoustical impedance and, especially, to detect slip-discontinuity. It is possible to show that *APOIS* method



can be used to detect adhesion quality between thin protective layer and solid base when the protective layer thickness  $h < 0,1$  mm. This method is very sensitive instrument for detection of the quality of acoustical contact, to detect small gas discontinuity in the gap of gluing solid surfaces. The mentioned before principles of APOIS using will be useful for development a new method of acoustical beam controlling.

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