REFLECTION OF LINEARLY POLARIZED SHEAR WAVES FROM THE STATISTICALLY ROUGH SURFACE OF CRACKS

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In practice of the US-control methods of irradiation of the structures by shear waves of not only SV-, but also SH-polarization are widely applied relative to the surface including cracks. The main task of the work was to estimate the parameters for field of transverse linearly polarized waves in the solid body on the statistically rough reflector in accordance to the orientation of the vector of linearly polarization of waves relative to its surface.

The considered design model (heuristic) for estimation of the index of shear wave reflection from the statistically rough surface is supposed to be two-dimensional one. The plane shear monochromatic wave falls on the surface at the angle θ_0, counted off from the normal towards average straight line z = 0 (Fig. 1) [1].

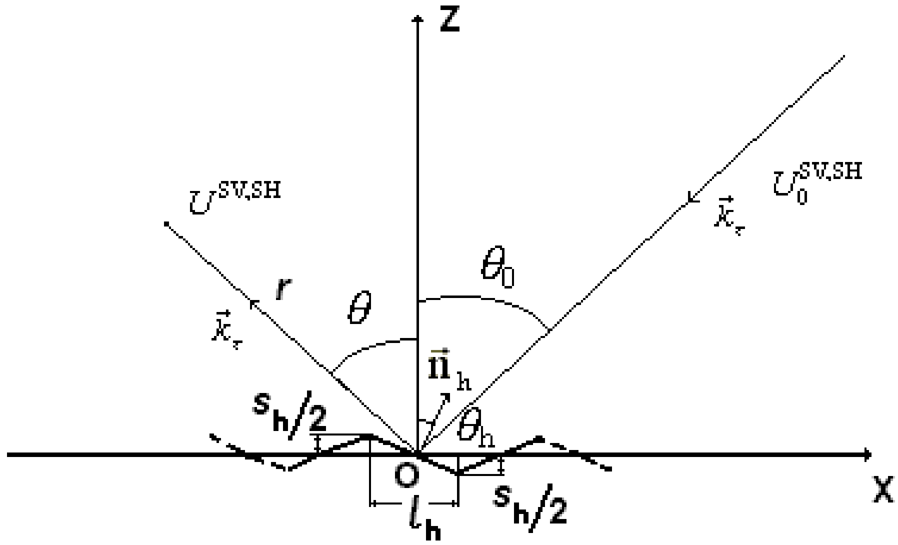

Fig.1.

It is supposed that inhomogeneous surface consists of scattering linearly elements with their altitude assignment s_h according to the normal law with the average quadratic deviation $\bar{s}_h$ and expected value m_{s_h} :

$$W(s_h) = \frac{1}{\sqrt{2\pi}\,\bar{s}_h} \exp\left[-\frac{(s_h - m_{s_h})^2}{2\bar{s}_h^2}\right] \qquad (1)$$

The element size along the average straight line is equal to l_h and connected to the height s_h by the some functional relation, the form of which in shape of regressive formula is defined by the experiment data and usually connects the average quadratic values $\bar{s}_h$ and $\bar{l}_h$ (correlation window). The linearly approximation of the next relationship $\bar{s}_h = q_h \bar{l}_h$, obtained in the work [1] is used for the calculations below, where the proportionality factor q_h, equal to 0,11 according to [1], could be some more or less of this value.

Orientation of the roughness element is defined by the angle θ_h of the normal slope $\vec{n}_h$ to its surface relative to axis OZ (Fig. 1) – normal to the average straight line (axis OX), at that $\tan\theta_h = s_h / l_h$.

The dimensions s_h and l_h are supposed to be small, as compared to the distance from the center of the scattering element to the wave recording point (r). The element center is located on the average straight line in the point O, that's why the distances by axis OZ to the element ends are equal to 0,5 s_h and -0,5 s_h. Definition of the amplitude of the wave scattered by the element is carried out in the two-dimensional approximation of Kirchhoff for acoustically soft reflector l_h long on the basis of the approach of B.A. Kruglov [2].

If the plane shear wave of vertical polarization (SV) with the amplitude U_0^{SV} falls on the element l_h long by axis OX at the angle θ_0 to vertical axis OZ (Fig. 1), then according to [2] the amplitude of the wave of the same polarization scattered at the angle θ (to OZ) for the plane task in the distance r at $k_\tau r >> 1, r >> l_h$ looks as follows:

$$U^{\mathrm{SV}} \cong U_0^{\mathrm{SV}} \frac{l_h}{\sqrt{\lambda_\tau r}} \cos\theta_0 \, I^{\mathrm{SV}}(\theta_0, \theta) \exp[i(k_\tau r - \pi/4)], \quad (2)$$

where $k_\tau = \dfrac{2\pi}{\lambda_\tau}$ - wave number of the shear wave with the length of $\lambda_\tau = \dfrac{c_\tau}{f_{\mathrm{P}}}$; (in Fig.1 $\vec{k}_\tau$ - wave vector in directions of direct and scattered waves)

$$I^{\mathrm{SV}}(\theta_0, \theta) = \frac{\sin[0{,}5 k_\tau l_h (\sin\theta \pm \sin\theta_0)]}{0{,}5 k_\tau l_h (\sin\theta \pm \sin\theta_0)} \begin{cases} \dfrac{\cos\theta \sqrt{n^2 - \sin^2\theta}}{\sin^2\theta + \cos\theta\sqrt{n^2 - \sin^2\theta}}, & n > \sin\theta \\[2ex] \dfrac{\cos\theta\sqrt{\sin^2\theta - n^2}}{\sqrt{\sin^4\theta + \cos^2\theta(\sin^2\theta - n^2)}}, & n < \sin\theta \end{cases}, \quad (3)$$

- scattering indicatrix of shear SV – surface element wave; $n = \dfrac{c_\tau}{c_l}$, c_l, c_τ - speeds of the compressional and shear waves in the environment; f_{P} - operating frequency. In (3) sign "+" is taken, if the scattered wave travel in the positive direction of axis OX (back scattering) and "-" – if in the negative direction (forward scattering).

For the plane shear wave of horizontal polarization (SH) with the amplitude U_0^{SH} the amplitude of the scattered wave in the same approximation has the form [2]:

$$U^{\mathrm{SH}} \cong U_0^{\mathrm{SH}} \frac{l_h}{\sqrt{\lambda_\tau r}} I^{\mathrm{SH}}(\theta_0,\theta)\exp[i(k_\tau r-\pi/4)], \quad (4)$$

$$I^{\mathrm{SH}}(\theta_0,\theta)=\frac{\sin[0,5k_\tau l_h(\sin\theta\pm\sin\theta_0)]}{0,5k_\tau l_h(\sin\theta\pm\sin\theta_0)}\cos\theta$$

Estimation of the reflection factors in the back and mirror directions offered in this work is based on calculation of the power flow of the reflected (scattered) waves spread from the rough surface averaged on possible values of the scattering element height s_h, connected with its dimension l_h and normal slope angle θ_h. It is supposed that changing of the amplitude of the scattered waves due to wave front spreading and attenuation are compensated due to absorption and scattering in the environment. In this case the amplitude reflection coefficient of SV-wave considering the formulae (2), (3) has the form of:

$$R_{\mathrm{o,з}}^{\mathrm{SV}} = \sqrt{\frac{\int |\frac{U^{\mathrm{SV}}\sqrt{\lambda_\tau r}}{U_0^{\mathrm{SV}}}|^2 \,\mathrm{W}(s_h)\,\mathrm{d}s_h}{\int l_h^2\,\mathrm{W}(s_h)\,\mathrm{d}s_h}} \cong$$

$$\cong \sqrt{\frac{\int l_h^2\{\cos\theta_0 I_{\mathrm{o,з}}^{\mathrm{SV}}[\theta_0,(\theta_0-\theta_h)]\}^2\,\mathrm{W}(s_h)\,\mathrm{d}s_h}{\int l_h^2\,\mathrm{W}(s_h)\,\mathrm{d}s_h}} \quad (5)$$

Normalizing of the reflection coefficient (5) is conditioned by the fact that in case of isotropic wave scattering (ordinary indicatrix) falling in the direction of axis OZ, then this coefficient is equal to 1. In formula (5) possibility of spreading of the elements $\mathrm{W}(s_h)$ is defined by formula (1); integration goes by the sphere of considered changes s_h (usually the size of $\bar{s}_h$ - from 3 $\bar{s}_h$ to + 3 $\bar{s}_h$); $(\theta_0 - \theta_h)$ – direction angle of spreading of the wave reflected from the element of surface roughness, counted off from the normal to the element surface (angle sign θ_h, specifying direction of the slope of the reflecting element, varied: : "+" – at $s_h > m_{s_h}$ and "-" – at $s_h < m_{s_h}$); indices "о (b), з (m)" mean scattering in back and mirror directions. In case of back scattering in the indicatrix formula (3) sign "+" is taken, and in case of mirror scattering – "-". By analogy the back scattering (normalized as is mentioned above) coefficient of SH wave considering formula (4) has the form of:

$$R_{\mathrm{o}}^{\mathrm{SH}} = \sqrt{\frac{\int |\frac{U^{\mathrm{SH}}\sqrt{\lambda_\tau r}}{U_0^{\mathrm{SH}}}|^2 \,\mathrm{W}(s_h)\,\mathrm{d}s_h}{\int l_h^2\,\mathrm{W}(s_h)\,\mathrm{d}s_h}} \cong$$

$$\cong \sqrt{\frac{\int l_h^2\{I_{\mathrm{o}}^{\mathrm{SH}}[\theta_0,(\theta_0-\theta_h)]\}^2\,\mathrm{W}(s_h)\,\mathrm{d}s_h}{\int l_h^2\,\mathrm{W}(s_h)\,\mathrm{d}s_h}} \quad (6)$$

By formulas (5), (6) and (1) dependences of relationships $\frac{R_{\text{о}}^{SV}}{R_{\text{з}}^{SV}}$ and $\frac{R_{\text{о}}^{SH}}{R_{\text{о}}^{SV}}$ were calculated from the value $\overline{s}_h$ for different values of the angle θ_0, operating frequency f_{P}, assemble average m_{s_h}, etc. In Figure 2 the dependence of $\frac{R_{\text{о}}^{SV}}{R_{\text{з}}^{SV}}$ on $\overline{s}_h$ is set forth for several values of the angle θ_0, $m_{s_h} = 2\overline{s}_h$, f_{P} = 2MHz and $\overline{s}_h = q_h \overline{l}_h$, $q_h = 0{,}11$, and in Figure 3 – similar dependence $\frac{R_{\text{о}}^{SH}}{R_{\text{о}}^{SV}}$ for the same parameters. While the height of the roughness element $\overline{s}_h$ increases, its horizontal dimension $\overline{l}_h$ also grows that leads to narrowing of direction diagram thereof, decreasing more the amplitude of wave back scattering than in the mirror direction, and this very fact leads to decreasing of the relationship $\frac{R_{\text{о}}^{SV}}{R_{\text{з}}^{SV}}$.

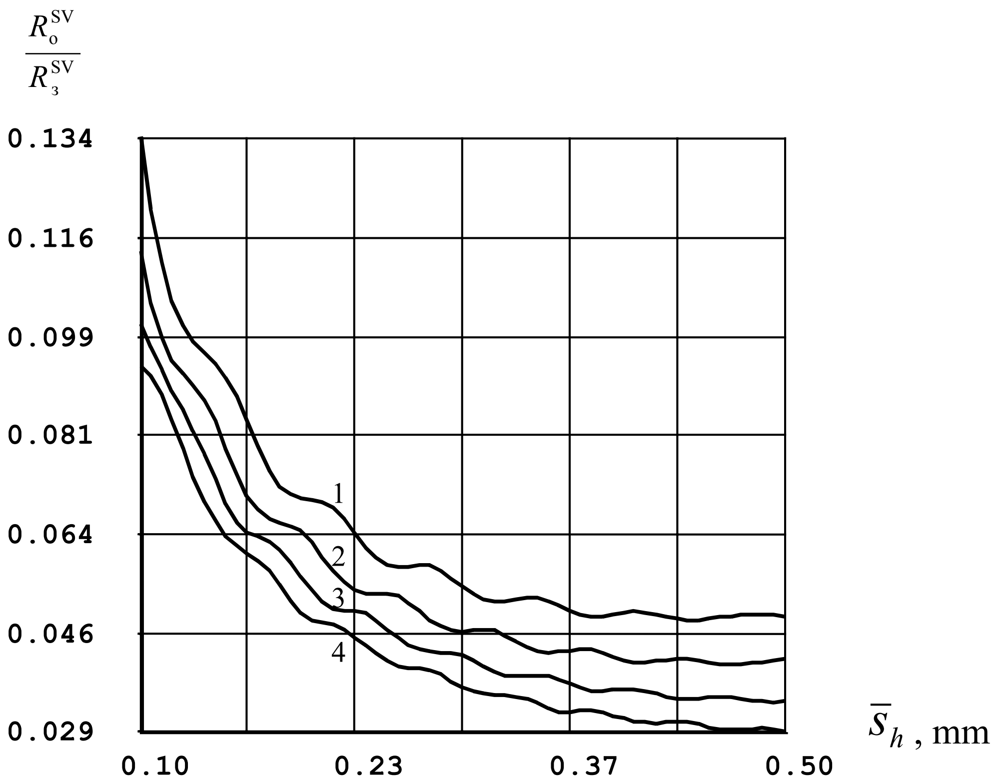

Fig.2. 1. θ_0 = 45°; 2. – ″ – 50°; 3. – ″ – 55°; 4. – ″ – 60°

It is clear from Figure 3 that changing of the parameter $\overline{s}_h$ affects the relationship $\frac{R_{\text{о}}^{SH}}{R_{\text{о}}^{SV}}$ insignificantly, this influence is more significant in the area of smaller values $\overline{s}_h$ for angles θ_0 = 45°; 50°, which changing, however, significantly affects this value within the whole design range of the increment caused by difference of the diagrams of elementary radiators' directions on the surface of the scattering element according to [2] for SV and SH waves (see formulae (3), (4)).

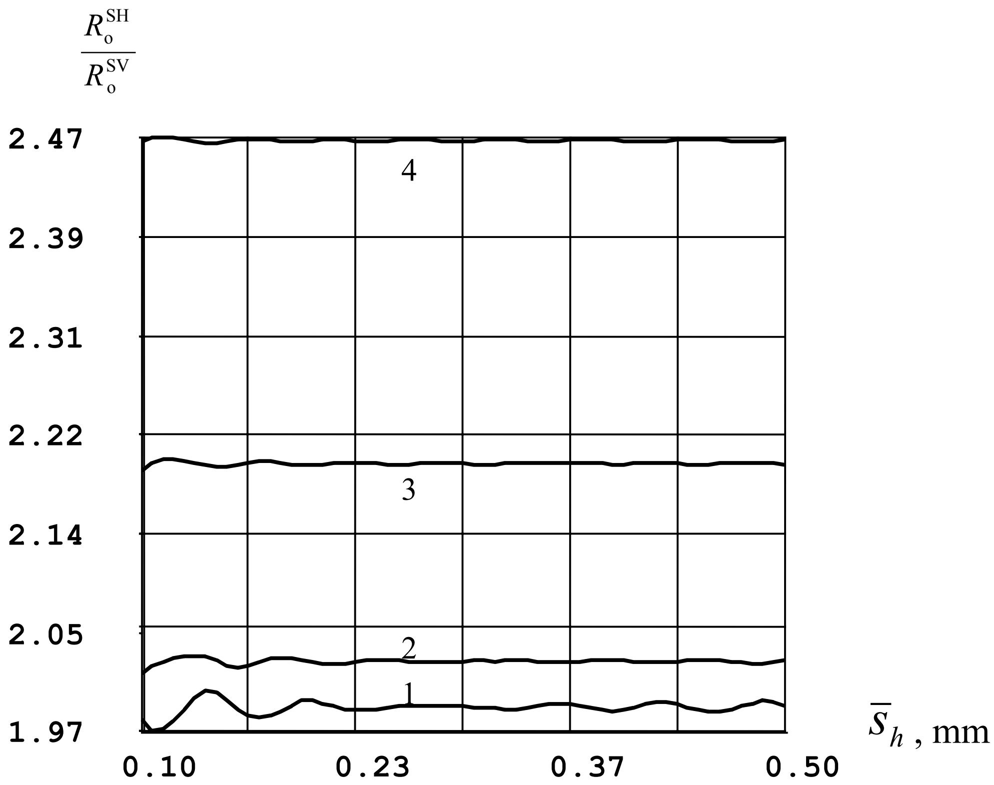

Fig.3. 1. θ_0 = 45°; 2. – ″ – 50°; 3. – ″ – 55°; 4. – ″ – 60°

Influence of the operating frequency changing on the relationship of the reflection coefficient of SV-wave $\frac{R_o^{SV}}{R_з^{SV}}$ is represented in Figures 4 and 5, where the values f_P are equal to 1,3 and 3 MHz respectively.

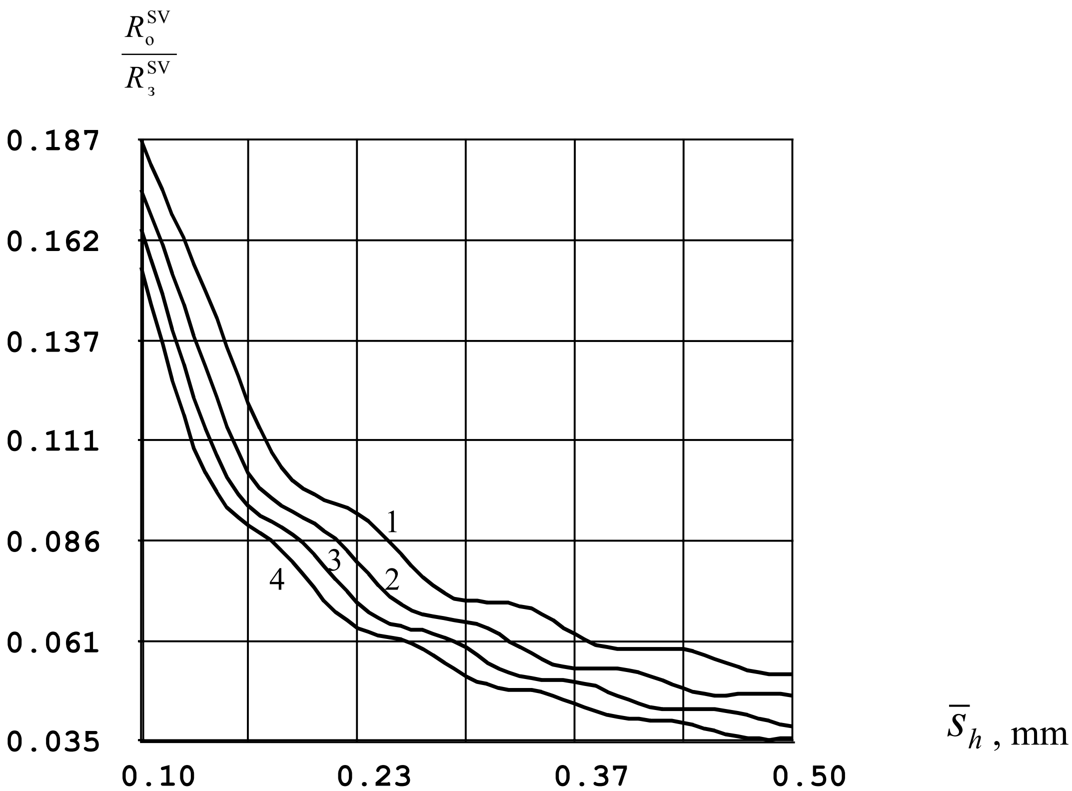

Fig.4. 1. θ_0 = 45°; 2. – ″ – 50°; 3. – ″ – 55°; 4. – ″ – 60°

Decreasing of f_P leads to extension of the diagram of the scattering element direction and reducing of the difference of amplitudes of the wave back and mirror scattering, increasing – to the inverse effect (compare the maximum values of dependences with the same numbers in Figures 2, 4 and 5). Certain increasing of the relationship $\frac{R_{o}^{SV}}{R_{\text{з}}^{SV}}$ in Figure 5 at $\bar{s}_h$ > 0,35 is conditioned by the growth of the back scattering coefficient.

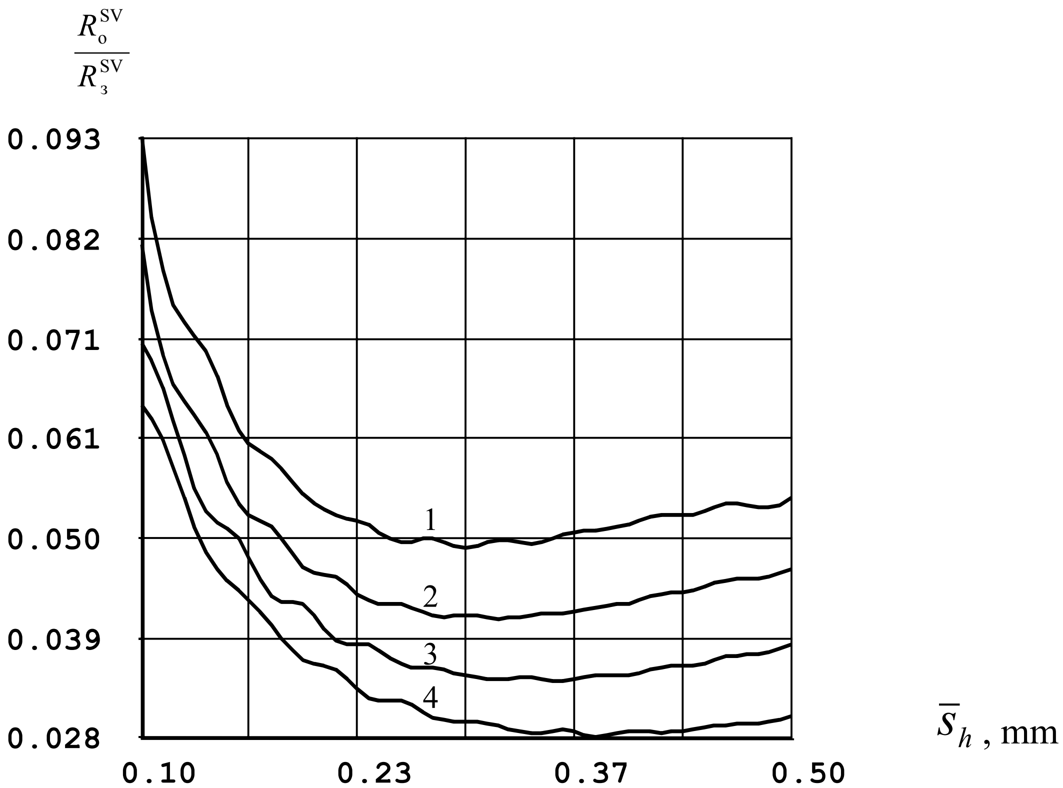

Fig. 5. 1. θ_0 = 45°; 2. – ″ – 50°; 3. – ″ – 55°; 4. – ″ – 60°

Changing of the frequency affects the ratio of the back reflection coefficients of SH – and SV – waves $\frac{R_{o}^{SH}}{R_{o}^{SV}}$ most insignificantly; insignificant balancing of dependences is observed while it increases (Fig. 6, f_P = 3 MHz).

Decreasing of the average dimension of the scattering element $\bar{l}_h$ at the same value $\bar{s}_h$, i.e. increasing of the parameter q_h, leads as during reducing of f_P to extension of the element direction diagram, and , therefore, to increasing of the ratio $\frac{R_{o}^{SV}}{R_{\text{з}}^{SV}}$ (Fig. 7, q_h = 0,14, the rest parameters are the same as for Figures 2 and 3).

Increasing of the average size of the element $\bar{l}_h$ leads to the inverse effect.

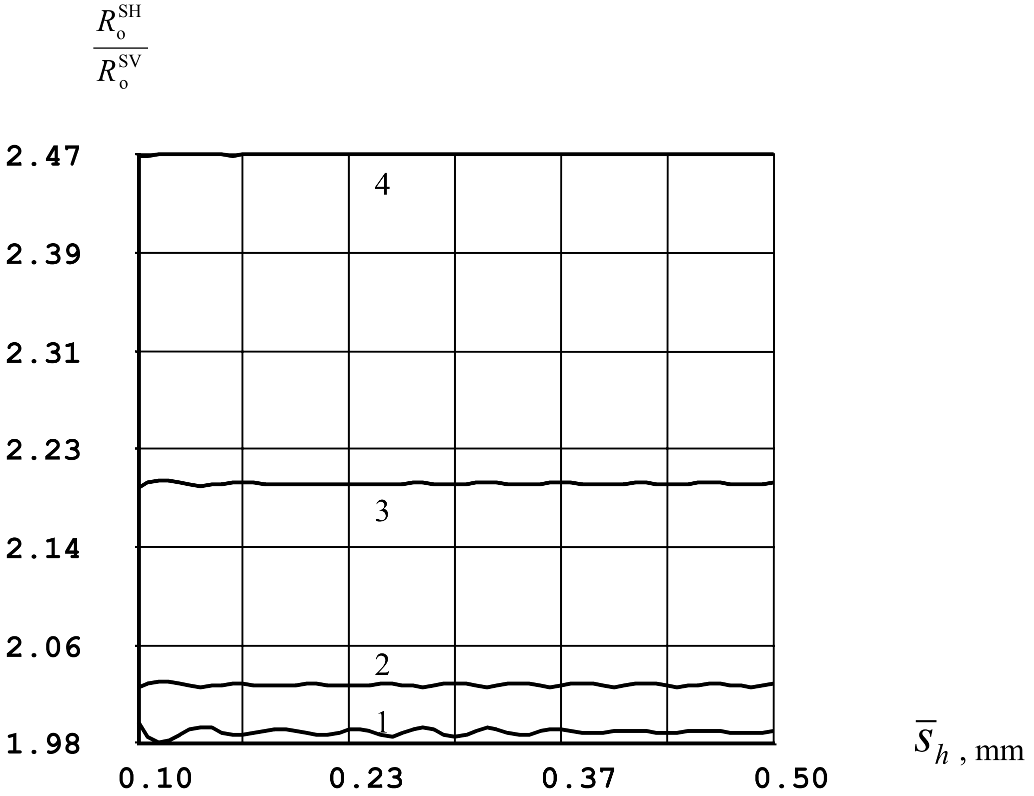

Fig.6. 1. θ_0 = 45°; 2. – ″ – 50°; 3. – ″ – 55°; 4. – ″ – 60°

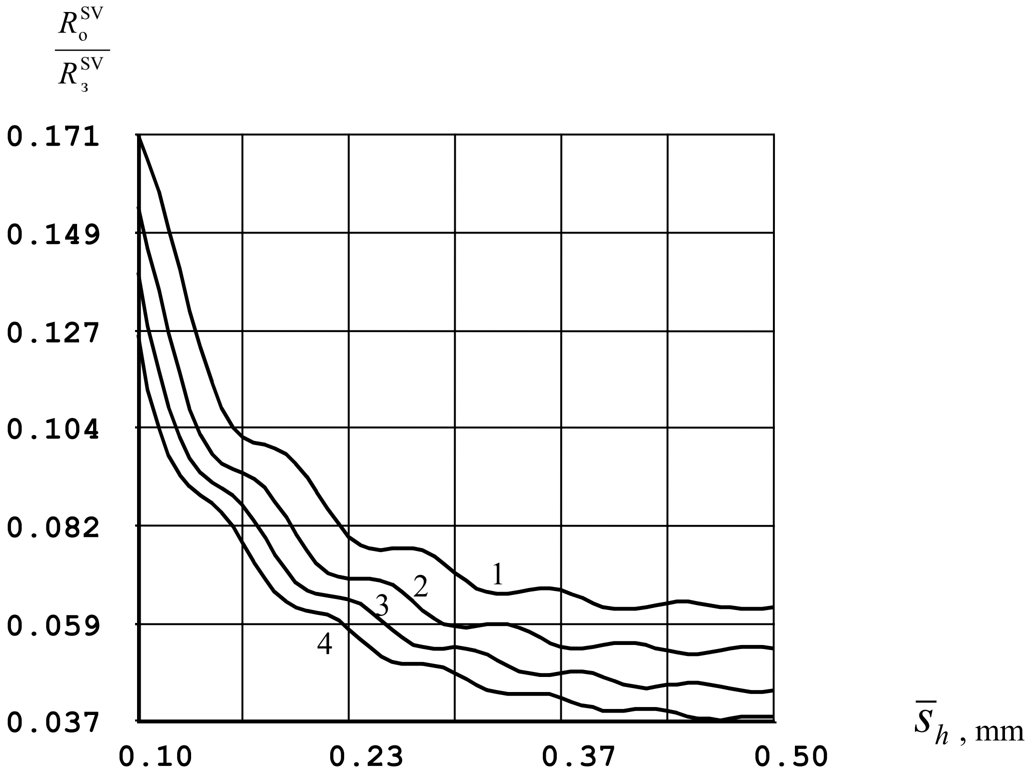

Fig. 7. 1. θ_0 = 45°; 2. – ″ – 50°; 3. – ″ – 55°; 4. – ″ – 60°

Increasing of the expected value m_{s_h}, i.e. the number of values with a large height s_h and, respectively, greater size of the scattering element l_h with narrower direction diagram leads to decreasing of the ration $\frac{R_{o}^{SV}}{R_{3}^{SV}}$, and decreasing of m_{s_h} - to growth of this ratio. Influence of changing of this parameter on dependence of the ratio $\frac{R_{o}^{SH}}{R_{o}^{SV}}$ on $\bar{s}_h$ is insignificant.

As physical crack models the fractures surfaces of samples of type CT-3 (90x90mm) can be used applied for estimation of brittle failure in case of statistical loading during the bending test. The numeric values of estimations of roughness spreading parameters in two orthogonal directions (with the interval of 0,25mm) were analyzed on the personal computer and compared to the similar estimation for real cracks. Such comparison demonstrated that statistical characteristics of roughs near the fracture surface are quite close to the similar characteristics of real splits [1].

The experiment was carried out on the triangle-section sample with the vertex angle of 80°. At that the incidence angle on the fracture surface made 50° and corresponded to real conditions and parameters of control of weld joints. The probe with the quartz y-cut plate was used for the frequency of 2,0 MHz, radiating linearly polarized waves and a special adjusting device. In each point the amplitudes of the reversal signal A_{rev} and signal A^1_3 reflected from the rough surface were measured, as well as from the opposite smooth plane at different orientation of the polarization vector simulating ultrasonication by SV- and SH- waves, as well as under the intermediate angle of 45°. As ultrasonic attenuation in the sample material heavily depended in the orientation of the wave polarization vector, normalizing of the amplitudes for definition of the ratio of wave scattering coefficients of different polarization was carried by supporting mirror signals from the smooth parts on the same sample. The results of experimental surveys (with theoretical estimation in the compared cases) are represented in the table.

Table

Characteristics of signals at different orientation of the polarization vector

Surface parameters	$\bar{s}_h$ = 0,23 mm; $\bar{l}_h$ = 1,63 mm P_R = 0,69			$\bar{s}_h$ = 0,36 mm; $\bar{l}_h$ = 3,2 mm P_R = 1,075		
Wave polarization	A_{rev} / A^1_3	ν (A)	ρ (A_{rev} / A^1_3)	A_{rev} / A^1_3	ν (A)	ρ (A_{rev} / A^1_3)
SH, A_{rev}		0,25			0,22	
SH, A^1_3	0,24	0,12	-0,94	0,24	0,12	0,30
45°, A_{rev}		0,064			0,25	
45°, A^1_3	0,17	0,11	-0,27	0,11	0,14	0,25
SV, A_{rev}	0,09	0,14		0,025	0,12	
SV, A^1_3	0,06 (theoretically)	0,11	-0,83	0,04 (theoretically)	0,14	0,39
$\frac{R_{o}^{SH}}{R_{o}^{SV}}$	1,8 2,0 (theoretically)			1,8 2,0 (theoretically)		
$\frac{R_{o}^{45}}{R_{o}^{SV}}$	1,5			1,1		

$\frac{R_3^{SH}}{R_3^{SV}}$	0,83			0,4		
$\frac{R_3^{45}}{R_3^{SV}}$	0,9			0,5		

In the table ν (A) – coefficient of the amplitude variant, P_R – Rayleigh parameter, ρ (A_{rev} / A^1_3) – cross-correlation coefficient. The results of the experiments prove theoretical ones in that the ratio of back reflection coefficients is significantly bigger than 1 that conditions better detection of cracks by SH-waves. For mirror-reflected signal this ratio is inverse and decreases proportionally to the Rayleigh parameter. This peculiarity is not proved theoretically, but probably it is connected with phenomenon of a wavy surface not considered in the estimation. Behavior of the dependence of the back and mirror scattering coefficients for SV-waves on the average quadratic deviation $\overline{S}_h$ (Rayleigh parameter) is adequately agreed with the results of estimations. The variation coefficient makes ν (A^1_3) < ν (A_{rev}) that proves greater statistic stability of the mirror signal. This a priori defines a smaller error of the equivalent area changing by the mirror signal.

The mutual correlation coefficients are weakly connected with polarization of waves ρ (A_{rev} / A^1_3), but strongly depend on the Rayleigh parameter P_R .

1. Scherbinskii V.G. Research of statistical parameters of roughs and reflection characteristics of cracks of weld joints. –Defektoskopiya, 1985, №11, pp. 9-20.
2. Kruglov B.A. On angle scattering characteristics by acoustically soft disk in the solid body of plane monochromatic compressional and shear SV- and SH- waves. – Matters of material science, 2001, №4, pp.66-77.