Regularized eddy current tomographic reconstruction based on finite element forward model

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Abstract

Eddy current tomography is a nondestructive evaluation technique used for the characterization of metallic components. It is an inverse problem acknowledged as difficult to solve since it is both ill-posed and nonlinear. Our goal is to derive an inversion technique with improved trade-off between quality of the results, computational requirements and ease of implementation. This is achieved by fully accounting for the nonlinear nature of the forward problem by means of a system of bilinear equations obtained through a finite element model of the problem. The bilinear character of equations with respect to the electric field and the relative conductivity is advantageously exploited through a simple contrast source inversion-type scheme. The ill-posedness is dealt with through the addition of regularization terms to the criterion, the form of which is determined according to computational constraints and the piecewise constant nature of the medium. Therefore an edge-preserving functional is selected. The performance of the resulting method is illustrated using 2D synthetic data examples.

1 Introduction

Eddy current nondestructive evaluation is used to characterize surface flaws within metal structures. It is used for flaws whose depth is too small to be efficiently detected by means of ultrasound techniques. Data are acquired through measurements of the impedance variation of a coil displaced in the neighborhood of the *object under test* (OUT). The goal is to use data in order to evaluate the relative conductivity of elements of discretized metallic parts of nuclear power plant with a triangular mesh. This problem is nonlinear and ill-posed which makes it difficult to solve [1].

Classical ways to deal with this kind of problem need a forward model to describe electromagnetic phenomena. In the diffraction tomography case, the forward model is most often based on an integral formulation of the problem through a resolution of the Helmholtz using a point-collocation-type approach, e.g. the method of moment, based on Green fucntions [2]. Only the discretization of the OUT is necessary for this approach. Unfortunately, Green functions are only known for canonical shapes (free space, tube or plate) and adapting them to other geometries may be difficult [3]. Furthermore, Green matrices obtained with a discretization of the exact solution of the Helmholtz equation through the *method of moments* (MoM) [3, 4], are dense and their calculation is relatively complicated. Other modeling methods such as *finite differences* (FD) [5] or *finite elements* (FE) [6] can be used. These methods are more flexible and can be adapted more easily to different geometries as compared to integral equations. Furthermore they lead to sparse matrices in the description of the forward problem. In a previous paper [7], FD [8] were resorted because they are easier to implement and have already been used for diffraction tomography [5]. Nevertheless, a 2D-FE model for eddy current tomography can be turned into a 3D model and offers the possiblity to discretize accurately where it is necessary: around the coil in our case. Therefore, in this paper a FE approach is used for the forward model.

For inversion purpose, a classical approach is to minimize the error between measurements and the output of the forward model using a local descent scheme [6]. Unfortunately, many resolutions of the forward model are necessary, which becomes a computational burden. Another classical and elegant approach is the *modified gradient method* (MGM) [9] and its derived versions such as *constrast source inversion* (CSI) [5, 10] and *multiplicative regularized contrast source inversion* (MRCSI) [11] in the case of diffraction tomography. MGM/CSI-type methods avoid the resolution of the forward model and take advantage of the bilinear equations with respect to the conductivity and electromagnetic variables. They use an alternate minimization of a criterion with respect to the electromagnetic variable and the relative conductivity, *i.e.,* the contrast.

The goal of this paper is to adapt the MGM/CSI-type scheme to the FE forward model in the case of eddy current tomography to derive an inverse method with a good trade-off between quality of the results, ease of implementation and computational needs. Therefore, we begin by a study of the 2D case even though other methods have been already proposed in the 3D case. A forward model is proposed with a bilinear structure for an efficient minimization. Furthermore, our contribution also concerns the use of an additive edge-preserving regularization to overcome the ill-posedness of the problem. Finally, the method takes advantage of the *a priori* information that the constrast in the metal plate is comprised between 0 and 1. Illustrations of the efficiency of the approach are presented through inversion results obtained on 2D synthetic data.

2 Forward model

Data about the flaw were acquired from measurements of the impedance variation of a moving coil displaced above the OUT. A total of L different positions of the coil were considered. A schematic of the discretized domain is presented in Figure 1.

Our goal is to propose a forward model with a simple algebraic structure that links the coil impedance variation $\Delta Z = \{\Delta Z_\ell\}_{\ell=1,\dots,L}$ to the contrast in the plate. Let us define the contrast vector $\boldsymbol{x} \in \mathbb{R}^{N_e}$ containing the relative conductivity values of the domain Ω

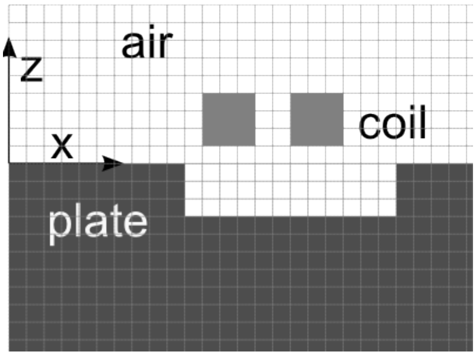


Figure 1: Data acquisition

divided into N_e elements:

$$\boldsymbol{x}(i) = (\sigma_0 - \boldsymbol{\sigma}(i))/\sigma_0, \quad i = 1, \ldots, N_e \ ,$$

where $\boldsymbol{\sigma}(i)$ is the conductivity of the i-th element.

This contrast vector enters in the electromagnetic formulation used for computing the electric field in the domain Ω for a given flaw $\boldsymbol{x}$. The electric field is required in order to compute the impedance variation of the coil.

In this work, the 2D transverse magnetic case is assumed [2], *i.e.,* only one component of the electric field (E) is nonzero in the domain Ω, and this component is perpendicular to the cross-section of the plate. The domain itself is divided into three distinct sub-domains: D_1, the air, e_2, the plate with the flaw, and E_ℓ, the coil at position ℓ.

The medium is considered as non magnetic, *i.e.,* the magnetic permeability in the whole domain Ω is equal to the magnetic permeability in vacuum μ_0. Furthermore the permittivity ϵ is uniform in the domain and equal to the permittivity of vacuum ϵ_0. We also define ω as the pulsation of the current in the coil, and we consider that the flaw is described only by the set of elements in which the conductivity σ differs from σ_0, which represent a perfect plate. With these assumptions, the Helmholtz equation for a position ℓ of the sensing coil can be written as [2]:

$$\Delta E_\ell(x,z) + k^2(x,z)\, E_\ell(x,z) = j\omega\mu_0 J_\ell(x,z) \tag{1}$$

$$k^2(x,z) = \omega^2\mu_0\epsilon_0 - j\omega\mu_0\sigma(x,z)$$

$$J_\ell(x,z) = \begin{cases} \pm J_c & \text{if } (x,z) \in D_\ell \\ 0 & \text{otherwise} \end{cases}$$

In these equations, $E_\ell(x,z)$ is the electric field at point (x,z), $J_\ell(x,z)$ is the current density, and $k^2(x,z)$ is the wave number. The coil is assumed to be of stranded type (many turns of fine wire), so its current density can be assumed uniform and equal to J_c.

Taking into account the conductivity and the frequency, displacement currents can be neglected in Equation (1), so the term $\omega^2\mu_0\epsilon_0$ is omitted in the remaining of this work [2]. Let us define the contrast function $c(x,z)$:

$$c(x,z) = (\sigma_0 - \sigma(x,z))/\sigma_0$$

The following approximated Helmholtz equation is obtained:

$$\Delta E_\ell(x,z) - qE_\ell(x,z) + q\sigma_0 c(x,z) E_\ell(x,z) = qJ_\ell(x,z) \tag{2}$$

$$q = j\omega\mu_0$$

in which the bilinear nature of the equation with respect to the conductivity and the electric field appears. We will take advantage of this feature for the inversion.

In order to solve this equation numerically, the approach chosen here is based on the discretization of the weak form (FE) of the differential equation. Details can be found in [12]. The Galerkin formulation is adopted, *i.e.,* we choose the test functions as identical to the base functions. First order Lagrange interpolation functions are used to approximate the electric field. Classical Dirichlet boundary conditions are applied, the field being assumed as zero on the boundary Γ of the domain Ω. It yields the following coupling equation:

$$\boldsymbol{M}\boldsymbol{e}_\ell + \boldsymbol{K}_x\boldsymbol{e}_\ell = \boldsymbol{f}_\ell \tag{3}$$

where $\boldsymbol{M} \in \mathbb{C}^{N_n\times N_n}$ is a constant matrix, $\boldsymbol{e}_\ell \in \mathbb{C}^{N_n}$ is the vector containing all the values of the electric field in the domain for a given position ℓ of the coil, and $\boldsymbol{f}_\ell$ is the vector containing the excitation, *i.e.,* the current density in the coil. The contrast enters in the $\boldsymbol{K}_x$ matrix. Let us denote $\boldsymbol{e} = \{\boldsymbol{e}_\ell\}_{\ell=1,\ldots,L}$.

Given a flaw fully described by the vector $\boldsymbol{x}$, the expression of ΔZ_ℓ can be found in [13]:

$$I_0^2\Delta Z_\ell = -\int_{D_\ell} E_\ell(x,z)\, J_c \mathrm{d}x\mathrm{d}z = -\int_{\Omega_f} E_{0\ell}(x,z)\, J_{d\ell}(x,z)\,\mathrm{d}x\mathrm{d}z \tag{4}$$

$$J_{d\ell}(x,z) = (\sigma(x,z) - \sigma_0)\, E_\ell(x,z)$$

where Ω_f is the flaw, *i.e.,* $\Omega_f = \{(x,z) \in D_2 \backslash \sigma(x,z) \neq \sigma_0\}$ and $E_{0\ell}(x,z)$ is the electric field in the domain Ω when there is no flaw. Let us define I_0 the current intensity and the S section of the coil :

$$I_0 = J_C S$$

Equation (4) is discretized for each position ℓ of the coil:

$$\Delta Z_\ell = \frac{1}{qI_0^2}\sigma_0 \boldsymbol{e}_{0\ell}^t (\boldsymbol{K}_{x0} - \boldsymbol{K}_x)\, \boldsymbol{e}_\ell \tag{5}$$

where $\boldsymbol{e}_{0\ell}$ is the electric field in the domain for the position ℓ above the device without any flaw. We also define $\boldsymbol{K}_{x0}$ as the matrix $\boldsymbol{K}_x$ for the case $\boldsymbol{x} = \boldsymbol{x}_0$, *i.e.,* the relative conductivity vector of the domain without flaw.

The final model to be used with the inversion algorithm is therefore described by Equations (3) and (5). It can easily be seen that is has the desired algebraic bilinear structure.

3 Inverse problem

The purpose of the inverse problem is to estimate the shape of the flaw from given data, *i.e.,* to find $\boldsymbol{x}$ from ΔZ. However, in such a case, the problem is ill-posed [1]: uniqueness and stability of the solution are not ensured.

A classical way to solve the inverse problem is to minimize a nonlinear least squares criterion based on the difference between ΔZ and the output of the forward model, which only depends on the contrast $\boldsymbol{x}$ [6]:

$$J_{NL}\left(\boldsymbol{x}\right)=\sum_{\ell}\left\|\Delta Z_{\ell}-\frac{1}{qI_0^2}\sigma_0\boldsymbol{e}_{0\ell}^t\left(\boldsymbol{K}_{x0}-\boldsymbol{K}_x\right)\left(\boldsymbol{M}+\boldsymbol{K}_x\right)^{-1}\boldsymbol{f}_{\ell}\right\|^2$$

The minimization is usually performed *with respect to* (w.r.t.) the contrast $\boldsymbol{x}$ thanks to a local optimization algorithm. However the computation of the gradient requires the resolution of the forward model, which is computationally costly.

Other classical approaches to solve such a problem are MGM/CSI-type methods [11]. These methods take advantage of the bilinear nature of the problem. They minimize the weighted errors of the observation and coupling equations, respectively denoted J_1 and J_2. The minimized criterion is written as follows:

$$J_1\left(\boldsymbol{x},\boldsymbol{e}\right)=\sum_{\ell}\left\|\Delta Z_{\ell}-\frac{1}{qI_0^2}\sigma_0\boldsymbol{e}_{0\ell}^t\left(\boldsymbol{K}_{x0}-\boldsymbol{K}_x\right)\boldsymbol{e}_{\ell}\right\|^2$$

$$J_2\left(\boldsymbol{x},\boldsymbol{e}\right)=\sum_{\ell}\left\|\boldsymbol{M}\boldsymbol{e}_{\ell}+\boldsymbol{K}_x\boldsymbol{e}_{\ell}-\boldsymbol{f}_{\ell}\right\|^2$$

$$J\left(\boldsymbol{x},\boldsymbol{e};\lambda\right)=J_1\left(\boldsymbol{x},\boldsymbol{e}\right)+\lambda J_2\left(\boldsymbol{x},\boldsymbol{e}\right) \tag{6}$$

There are several versions of MGM/CSI-type methods. The most common method calculates λ at each iteration with the expression detailed in [5, 10]. Furthermore, the minimized criterion may be either regularized with a multiplicative term [11] or additive term [14].

In all cases, two steps are repeated alternately:

1. minimization of the criterion w.r.t. the electromagnetic variable;

2. minimization of the criterion w.r.t. the contrast.

Our approach takes advantage of the bilinear nature of Equations (3) and (5) and is also based on the alternative minimization of the criterion $J\left(\boldsymbol{x},\boldsymbol{e};\lambda\right)$ in Equation (6).

Therefore, λ is maintained constant during the inversion as proposed in [14, 15]. Indeed, the empirical way to compute λ used in [10] may provide degenerated solutions [14].

A regularization term $\phi(\boldsymbol{x};\beta,\gamma,\delta)$ is added in the criterion to overcome the ill-posedness of the problem. Additive regularization terms are more classical in the inverse problems methodology than multiplicative terms. The piecewise constant nature of the flaw is taken into account through an edge-preserving penalty function $\phi(\boldsymbol{x};\beta,\gamma,\delta)$ plotted in Figure 2. Furthermore, it is considered *a priori* that there are much more elements with a zero contrast than nonzero contrast. This is taken into account through a L_1-norm term in the regularization. The resulting regularization term is :

$$\phi(\boldsymbol{x};\beta,\gamma,\delta) = \beta\psi(\boldsymbol{D}\boldsymbol{x};\delta) + \gamma|\boldsymbol{x}|$$

$$\psi(u;\delta) = \sqrt{u^2+\delta^2}$$

where the $\boldsymbol{D}$ matrix corresponds to the finite difference operators between elements.

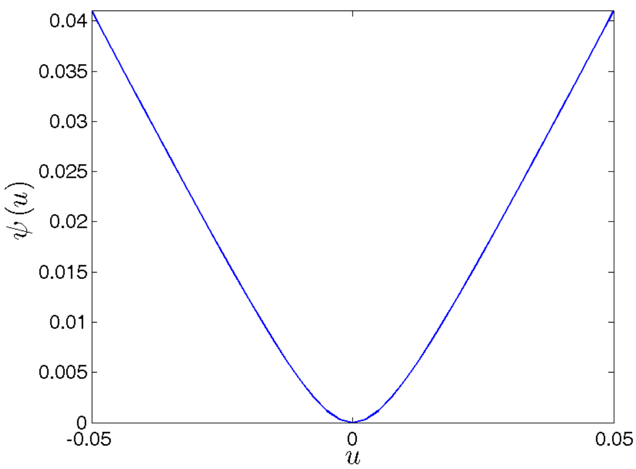


Figure 2: $\psi(u;\delta) = \sqrt{u^2+\delta^2}$

An asymptotical analysis of the function $\psi(\boldsymbol{x};\delta)$ shows that it penalizes low variations of $\boldsymbol{x}$ quadratically and high variations linearly. The hyperparameter β controls the importance of the *a priori* w.r.t. the observation of coupling equations. The hyperparameter δ tunes the transition between the linear/quadratic behavior of the penalty function near $u=0$. Finally, the hyperparameter γ serves to reduce the sparseness of the solution. Moreover, it is known *a priori* that $\sigma \in [0,\sigma_0]$, and therefore the contrast $\boldsymbol{x}$ belongs to $[0,1]^{N_e}$.

Our method is divided into two steps which are alternately repeated until convergence is reached, as described in Table 1. The implementation of the method is simple and takes advantage of the bilinear structure of the equations.

The initialization of the algorithm is as follows:

Computation of $\boldsymbol{e}_0$;
Initialization : $n \leftarrow 1$, $\left\{\boldsymbol{e}_\ell^{(0)}\right\}_{\ell=1,\dots,L}$ and $\boldsymbol{x}^{(0)}$;
repeat
  for $\ell = 1$ to L **do**
    $\boldsymbol{e}_\ell^{(n)} = \arg\min_{\boldsymbol{e}} J\left(\boldsymbol{x}^{(n-1)}, \boldsymbol{e}; \lambda, \beta, \gamma, \delta\right)$;
  end for
  $\boldsymbol{x}^{(n)} = \arg\min_{\boldsymbol{x}\in[0,1]^{N_e}} J\left(\boldsymbol{x}, \boldsymbol{e}^{(n)}; \lambda, \beta, \gamma, \delta\right)$;
  $n \leftarrow n+1$;
until convergence
Return $\boldsymbol{x}$.

Table 1: Algorithm of our approach

- initialization of the electric field: $\boldsymbol{e}^{(0)} = \boldsymbol{e}_0$ assuming that the total field in presence of a flaw is not very different from the incident field;
- initialization of the contrast: $\boldsymbol{x}^{(0)} = \boldsymbol{x}_0$.

The minimization step of the criterion w.r.t. the electric field $\boldsymbol{e}$ is performed through a linear *conjugate gradient* (CG) algorithm [15] given the quadratic nature of the criterion w.r.t. the electric field. The minimization at iteration n is initialized with $\boldsymbol{e}^{(n-1)}$ and truncated to speed up the algorithm.

The minimization of the criterion w.r.t. the contrast $\boldsymbol{x}$ is quite different. The criterion is not quadratic w.r.t. $\boldsymbol{x}$ but is still convex. The minimization of $J(\boldsymbol{x}, \boldsymbol{e}; \lambda, \beta, \gamma, \delta)$ w.r.t. $\boldsymbol{x}$ is performed thanks to a quasi-Newton algorithm limited memory boundary constrained called L-BFGS-B [16]. As previously, the minimization at iteration n is initialized with $\boldsymbol{x}^{(n-1)}$ to ensure the decrease of the criterion.

Finally, a stopping criterion is introduced w.r.t. the relative variation of the contrast between two iterations, *i.e.*, the convergence is reached at iteration n when

$$\left\|\boldsymbol{x}^{(n)} - \boldsymbol{x}^{(n-1)}\right\|^2 / \left\|\boldsymbol{x}^{(n)}\right\|^2 \leqslant \epsilon = 10^{-10}.$$

4 Results

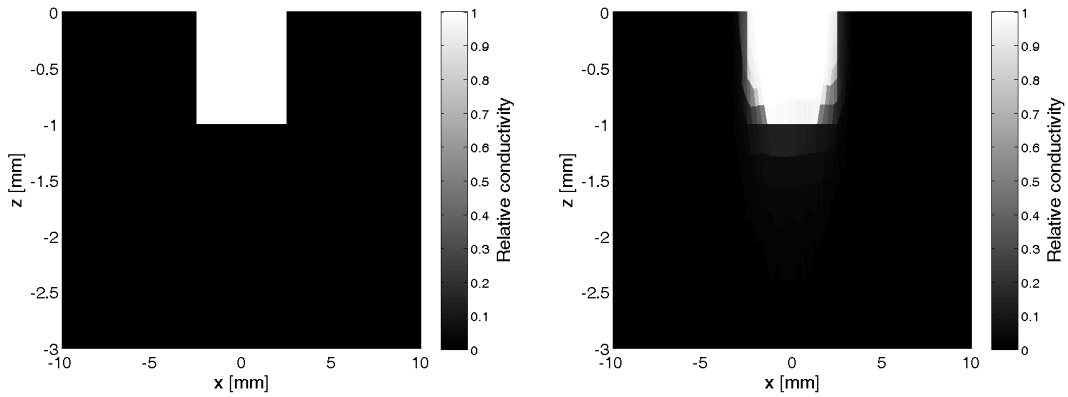
Here the performance of the method presented in the previous section with 2D synthetic examples is illustrated. Measurements of the coil impedance variation ΔZ^* for a known flaw $\boldsymbol{x}^*$ are simulated.

In order to simulate a real eddy current sensing coil, the square coil had the following geometry: inner radius 1 mm, outer radius 3 mm, lift-off 0.2 mm, height 1 mm and current intensity $I_0 = 1$ A. We used $L = 71$ positions for the coil.

The inverse crime [17] is committed when the model used to generate the data is the same than the one used for the inversion. In order to avoid it, synthetic data were generated using a commercial software based on the finite element modeling of the Helmholtz equation (COMSOL Multiphysics). It uses a formulation based on the magnetic potential vector, as opposed to our model, which solves for the electric field.

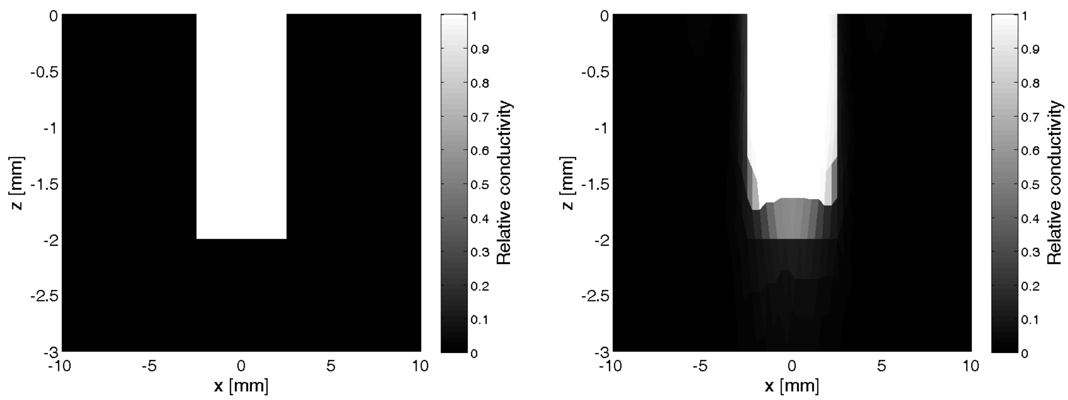
Data were simulated for several flaws with different shapes. Here, we present only two typical flaws to demonstrate the general efficiency of the method. The first flaw is 1 mm deep and 5 mm long. The second flaw is 2 mm deep and 5 mm long.

A unique set of hyperparameters that yields a good trade-off in term of quality of results for several flaws was chosen: $\lambda = 10^{13}$, $\beta = 10^{-4}$, $\gamma = 10^{-5}$ and $\delta = 10^{-2}$.



(a) Simulated flaw (b) Reconstructed flaw

Figure 3: Flaw 1: Simulated flaw (left) - Reconstructed flaw (right).



(a) Simulated flaw (b) Reconstructed flaw

Figure 4: Flaw 1: Simulated flaw (left) - Reconstructed flaw (right).

Reconstruction results are presented in Figures 3 and 4 for the flaws 1 and 2. It can be observed that even for deep flaws the method provided good results. Indeed, for a 2 mm deep flaw, the results are still good even if the skin depth is around 0.7 mm.

Let us indicate that each complete reconstruction required between one and two hours to reach convergence.

5 Conclusion

The goal of the work presented in this paper was to reconstruct the shape of flaws in metallic parts of nuclear power plants using data from eddy-current sensors. A 2D approximation was used as a first step. The approach presented showed good results when used with synthetic data, and it was easy to implement thanks to the bilinear nature of the equations.

The final goal of our work is to use this kind of approach for 3D eddy current tomography with real data. The 3D problem adds several practical challenges to what was presented here, namely meshing issues due the the displacement of the 3D sensing coil, and also memory issues. The use of a boundary integral / finite element approach is expected to help by removing the need of meshing the air and the coil domains, and thus reduce the number of unknowns in the problem.

Among other topics to be explored are means to increase the convergence speed and techniques to save memory. For instance, the electric field for different positions of the coil could be computed in parallel, independently from each other. Moreover, preconditioning techniques will be introduced to speed up convergence of the minimizations.

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