SIMPLE AND PRECISE INTERPOLATING FORMULAS FOR CALCULATING THE CENTRAL DEMAGNETIZATION COEFFICIENT FOR FERROMAGNETIC BODIES

S.G. Sandomirsky (United machine-building institute of the NAS of Belarus, Republic of Belarus, Minsk)

A solid and a hollow cylinder, a bar of an arbitrary profile, a plate (Fig.1) made of materials with high (μ >> 1) and low ($\mu \approx 1$) magnetic permeability are the physical models for many industrial and electro technical objects.

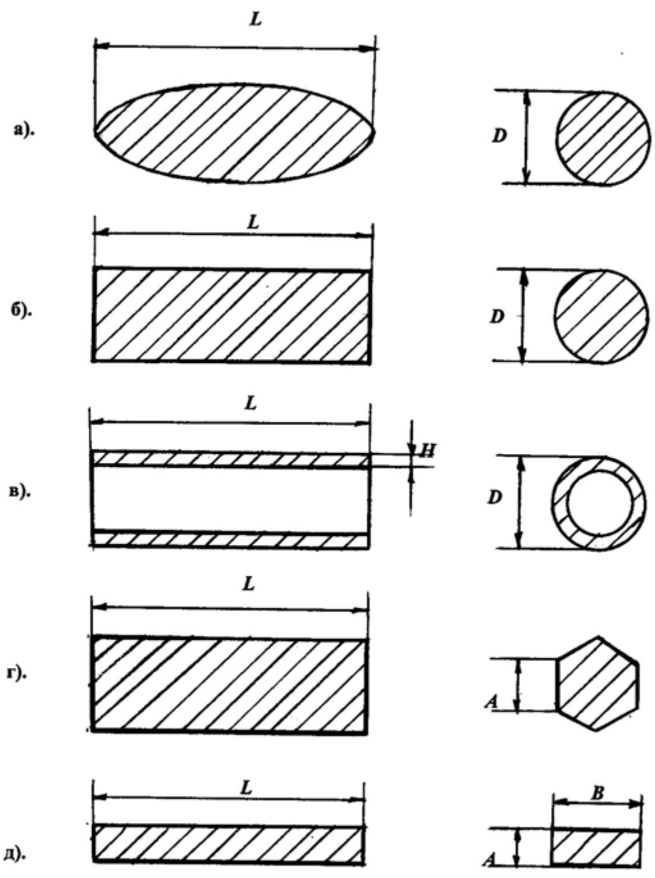

Figure 1. Longitudinal and transverse sections of the considered ferromagnetic bodies with notations of their geometrical measures.

The parameter, which determines the processes of magnetization and functioning of these objects, is their central demagnetization coefficient *N*.

On basis of the author's investigations [1 – 3] and analysis of the known theoretical and experimental results, the simple and precise interpolating formulas are recommended to use for calculating *N*:

Table 1

The shape of the body (figure)	Formula for calculation	Notations, parameters' variation ranges
Rotation ellipsoid	$N = Э(\lambda)$ $\lambda = \dfrac{L}{D}$	$Э(\lambda) = \dfrac{1}{1-\lambda^2}\left[1 - \dfrac{\lambda}{\sqrt{1-\lambda^2}}\arccos\lambda\right]$, $0 \le \lambda < 1$ $Э(\lambda) = \dfrac{1}{\lambda^2-1}\left[\dfrac{\lambda}{\sqrt{\lambda^2-1}}\ln\left(\lambda+\sqrt{\lambda^2-1}\right)-1\right]$, $\lambda > 1$
Cylinder, magnetized longitudinally	$N \approx Э(\lambda)\cdot k(\lambda)$ $\lambda = \dfrac{L}{D}$	$k(\lambda) = \dfrac{1+2.35\cdot\ln(1+0.137\cdot\lambda)}{1+2.28\cdot\ln(1+0.284\cdot\lambda)}$, $\mu >> 1$
		$k(\lambda) = \dfrac{1}{1+2.15\cdot\ln(1+0.326\cdot\lambda)}$, $\mu \approx 1$
Polyhedron (n - number of faces)	$N \approx Э(\lambda_{ЭФ})\cdot k(\lambda_{ЭФ})$	$\lambda_{ЭФ} = \dfrac{L}{A}\cdot\sqrt{\dfrac{\pi\cdot tg\frac{\pi}{n}}{n}}$, $0 \le \lambda_{ЭФ} < \infty$
Plate, magnetized along the side L	$N \approx Э(\lambda_{ЭФ})\cdot k(\lambda_{ЭФ})$	$\lambda_{ЭФ} = \dfrac{L}{2}\cdot\sqrt{\dfrac{\pi}{A\cdot B}}$, $0 \le \lambda_{ЭФ} < \infty$
Hollow cylinder, magnetized longitudinally	$N \approx h\cdot(2-h)\cdot Э(\lambda)\cdot k(\lambda)$	$\lambda = \dfrac{L}{D}$; $h = \dfrac{2\cdot H}{D}$, $0.2 \le \lambda < \infty$, $0 \le h \le 1$
	$N \approx \dfrac{1}{1+\beta\cdot\lambda^{\varepsilon}}$	$\varepsilon = 0.3075\left[1+\left(\dfrac{1.41}{h(2-h)}\right)^{-1}\right]$; $\beta = 5^{\varepsilon}\left[\dfrac{1.41}{h(2-h)}-1\right]$, $\lambda \le 0.3$
Hollow cylinder, magnetized orthogonally to the axis	$N_{\perp} \approx \dfrac{h}{2}\left[1 - Э(\lambda)\cdot k(\lambda)\right]$	$0 \le \lambda < \infty$, $0 \le h \le 1$

The experimental and theoretical results, which support the proposed recommendations, are presented in the following Supplements.

Supplement 1: Rotation ellipsoid. For calculating the demagnetization coefficient N of a rotation ellipsoid with axes ratio λ, which is magnetized along the rotation axis, the following equalities were obtained [4]:

$$N = Э(\lambda) \quad , \tag{1}$$

where

$$Э(\lambda) = \frac{1}{1-\lambda^2}\left[1 - \frac{\lambda}{\sqrt{1-\lambda^2}}\arccos\lambda\right] \quad \text{for} \quad 0 \le \lambda < 1 , \tag{2}$$

$$Э(\lambda) = \frac{1}{\lambda^2-1}\left[\frac{\lambda}{\sqrt{\lambda^2-1}}\ln\left(\lambda+\sqrt{\lambda^2-1}\right)-1\right] \quad \text{for} \quad \lambda > 1 . \tag{3}$$

The interpolation of this precise and simple formula, not depending on μ of the material of the ellipsoid, by a more complicated approximate formula -- as introduced by the authors of [5] in order to create a “universal” formula for calculating the N of ellipsoids, cylinders, and plates – is, obviously, nonsensical.

According to the basic physical notions [4], for a sphere with $N = 1/3$ we obtain that if $\lambda > 50$, then [4]:

$$N \approx \frac{1}{\lambda^2} \cdot \left[\ln(2 \cdot \lambda) - 1\right] \qquad \text{for} \quad \lambda >> 1 \quad . \tag{3'}$$

The demagnetization coefficient N_∞ of rotation ellipsoid magnetized perpendicular to its rotation axis equals [4]:

$$N_\perp = \frac{1}{2}(1 - \text{Э}(\lambda)) \quad . \tag{3''}$$

Supplement 2: Solid cylinder, magnetized longitudinally. Analysis [1] of the most thorough and reliable investigations of N for cylinders [6 - 12] shows, that to calculate N of a solid cylinder made of materials with high magnetic permeability ($\mu >> 1$) and with a ratio λ of the length L to diameter D in the range of $0 \le \lambda \le 100$, an interpolating formula can be recommended . This formula was proposed by Warmuth K. [10]:

$$N \approx \text{Э}(\lambda) \cdot k(\lambda) \quad , \tag{4}$$

where

$$k(\lambda) = \frac{1 + 2.35 \cdot \ln(1 + 0.137 \cdot \lambda)}{1 + 2.28 \cdot \ln(1 + 0.284 \cdot \lambda)} \quad . \tag{5}$$

The Table 2 shows the experimental results [9, 11, 12 – 14] of measuring N of cylinders and the results of calculating N according to (4) in the range of variation λ ($0.8 \le \lambda < 50.4$). The calculation error is computed with the formula

$$\sigma,\% = \frac{N_{calculation} - N_{experiment}}{N_{experiment}} \times 100\% \quad . \tag{6}$$

The accuracy of the interpolation by (4) of the results of measuring N, obtained by the authors in the whole practically relevant variation range of λ, must be acknowledge to be high, especially considering that μ and λ cylinders and their magnetic homogeneity are not always reliably known in practice.

Table 2

Comparisons of the calculated according to (4) and the experimental data for N cylinders.

№ пп	λ	Data source ([k]-author)	Experiment	Calculation with (4)	σ, %	№ пп	λ	Data source ([k]-author)	Experiment	Calculation with (4)	σ, %
1	0.872	[11][1]	0.3311	0.312	-5.8	41	28.45	[13][1]	0.003119	0.002955	-5.3
2	0.9		0.3165	0.303	-4.3	42	29.32		0.00296	0.002815	-5.0
3	1.778		0.1439	0.154	+7.0	43	30	[9][1]	0.00279	0.002712	-2.8
4	1.7875		0.1477	0.153	+3.6	44		[9][3]	0.002722		-0.4
5	2.5		0.09852	0.103	+4.5	45		[9][5]	0.002897		-6.4
6	2.667		0.09066	0.095	+9.0	46		[9][6]	0.002849		-4.8
7	3.333		0.0684	0.071	+3.8	47	30.19	[13][1]	0.002841	0.002685	-5.5
8	4.0		0.05333	0.056	+5.0	48	31.02		0.002737	0.002569	-6.1
9	5.333		0.03521	0.038	+7.9	49	31.86		0.002634	0.00246	-6.6
10	10	[9][1]	0.016	0.0150	-3.2	50	32.7		0.002491	0.002358	-5.3
11		[9][2]	0.01481		+1.3	51	33.56		0.002395	0.00226	-5.6
12		[9][3]	0.0151		-0.8	52	34.39		0.002324	0.002171	-6.6
13		[9][4]	0.0172		-12.7	53	35.24		0.002204	0.002086	-5.4
14		[9][5]	0.0154		-3.0	54	36.07		0.002093	0.002008	-4.1
15		[9][7]	0.01552		-3.4	55	36.91		0.002013	0.001934	-3.9
16	11.56	[14][1]	0.0124	0.01257	+1.4	56	37.75		0.001958	0.001864	-4.8
17	15	[9][5]	0.008356	0.008141	-2.6	57	38.59		0.001862	0.001747	-3.5
18		[9][7]	0.008515		-4.4	58	39.45		0.001798	0.001733	-3.6
19	19.25	[9][1]	0.005738	0.005514	-3.9	59	40	[9][1]	0.001723	0.001694	-1.8
20	20	[9][1]	0.00539	0.005191	-3.7	60		[9][3]	0.001703		-0.5
21		[9][3]	0.00518		+0.2	61		[9][6]	0.001775		-4.6
22		[9][4]	0.00617		-15.8	62		[9][7]	0.001775		-4.6
23		[9][5]	0.005348		-2.9	63	40.30	[13][1]	0.001751	0.001674	-4.4
24		[9][6]	0.005371		-3.4	64	41.18		0.001743	0.001615	-7.3
25		[9][7]	0.005454		-4.9	65	42.02		0.001615	0.001562	-3.4
26	20.09	[13][1]	0.00534	0.005155	-3.5	66	42.86		0.001552	0.001512	-2.6
27	20.92		0.005061	0.004835	-4.5	67	43.70		0.001512	0.001464	-3.8
28	21.76		0.004759	0.004542	-4.6	68	44.54		0.001472	0.001418	-3.7
29	22.60		0.004504	0.004276	-5.1	69	45.38		0.001424	0.001375	-3.4
30	23.43		0.004178	0.004037	-3.4	70	46.22		0.001377	0.001334	-3.1
31	24.27		0.003979	0.003816	-4.1	71	47.23		0.001321	0.001287	-2.8
32	25	[9][1]	0.004078	0.003639	-10.7	72	48.23		0.001273	0.001243	-2.4
33		[9][2]	0.003796		-4.1	73	49.25		0.001241	0.001200	-3.3
34		[9][3]	0.00366		-0.6	74	50	[9][1]	0.00118	0.00117	-1.0
35		[9][6]	0.003788		-3.9	75		[9][2]	0.001225		-4.5
35		[9][7]	0.003883		-5.8	76		[9][3]	0.00118		-0.7
37	25.10	[13][1]	0.003764	0.003616	-3.9	77		[9][4]	0.00129		-9.2
38	25.94		0.003581	0.00343	-4.2	78		[9][6]	0.001218		-3.9
39	26.78		0.003438	0.003259	-5.2	79		[12][1]	0.0012		-2.5
40	27.56		0.003287	0.003111	-5.4	80	50.42	[9][8]	0.001205	0.001154	-4.2

Footnote. Authors for the experiments: [11][1] – Slivinskaya A.G.; [9][1] – Neumann and Warmuth K.; [9][2]- Steiblein und Schlechtweg; [9][3] – Warmuth K.; [9][4] –R.M. Bozorth; [9][5] - Burdin; [9][6] - Wurschmidt J. ; [9][7] – Tompson and Moss.; [9][8] – Rosenblatt M.A.; [13][1] – Wurschmidt J.; [12][1] – Burtsev G.A.; [14][1] – Dietz G.

Supplement 3: Bar of arbitrary profile, plates.

Authors of [7, 16, 13] suggest to compute N of plates with an arbitrary shape of profile using formulae for a solid cylinder, the sectional area of which equals the area S of the metal in the cross-section of the bar, orthogonal to the direction of the magnetizing field. Applying this approach, calculation of N can be accomplished for solid bars of length L with arbitrary shape of profile at their magnetization parallel to the side L according to the formula (4). Here, instead of λ , its "effective" value $\lambda_{Ef.}$ is to be used. The latter is computed with the formula:

$$\lambda_{Ef} = \frac{L}{2}\sqrt{\frac{\pi}{S}} \quad . \tag{7}$$

Fore solid plates with a rectangular profile with sides A , B :

$$\lambda_{Ef} = \frac{L}{2}\cdot\sqrt{\frac{\pi}{A\cdot B}} = \frac{\lambda'}{2}\cdot\sqrt{\frac{\pi}{\gamma}} \quad , \tag{7'}$$

where $\lambda' = \dfrac{L}{A}$; $\gamma = \dfrac{B}{A}$, For solid rectilinear n-gons with side A:

$$\lambda_{Ef} = \lambda'\cdot\sqrt{\frac{\pi\cdot tg\frac{\pi}{n}}{n}} \quad . \tag{7''}$$

Comparison of the results of calculating N for solid and composite (*) plates according to formulae (4) and (7) with the known from [9, 11, 15] results of measuring N for plates is presented in the Table 3. We can notice the reasonable accordance of the calculation with experiment, taking into account the posible methodological measuring errors. The results of calculations with (4) and (7) are shown to be correct for the limit cases of varying the dimensions of plates. This allows to recommend the formulae (4) and (7) for computing N of solid or composite plates.

Table 3

Comparison of the computed with (4), (7) and experimental values for N of solid and composite plates of rectangular profile.

№ п/п	Data source	Dimensions					$N_{exp.}$	Computation results		
		L, мм	B, мм	A, мм	γ	S, мм^2		λ_{Ef}, Calculation with (7)	N, Calculation with (4), (7)	σ,% Calculation with (6)
1	[11], Tabl. 4, № 4	281	40	20	2	800	0.3111	0.88	0.309	-0.7
2	[11], Tabl.4, № 1	35.3	35	35	1	1225	0.3143	0.894	0.305	-2.9
3	[11], Tabl.4, № 6	24.8	60	10	6	600	0.2682	0.897	0.304	+13.4
4	[11], Tabl.4, № 7	32.2	60	10	6	600	0.2212	1.165	0.240	+8.3
5	[11], Tabl.4, № 5	49.1	60	10	6	600	0.1138	1.776	0.154	+35.3
6	[11], Tabl.4, № 2	30	15	15	1	225	0.1432	1.777	0.154	+7.8
7	[11], Tabl.4, № 3	57	40	20	2	800	0.1265	1.786	0.153	+21.0
8	[9], Tabl.8, № 9	30	3	0.2	15	0.6	0.002085	34.32	0.002178	+4.5
9	[9], Tabl.9, № 5	30	3	0.2	15	0.6	0.00227	34.32	0.002178	-4.0
10	[9], Tabl.10, №2	297	10	10	1	56*	0.002284	35.17	0.002093	-8.4
11	[9], Tabl.10, №1	297	10	5	2	28*	0.001241	49.74	0.00118	-4.9
12	[9], Tabl.8, №8	49	3	0.2	15	0.6	0.000995	56.1	0.0009662	-2.9
13	[15], Tabl.2, №3	49.1	3	0.2	15	0.6	0.0009947	56.2	0.0009629	-3.2
14	[9], Tabl.9, № 6	30	3	0.06	50	0.18	0.000690	62.7	0.000801	+16.1
15	[9], Tabl.8, № 7	70	3	0.2	15	0.6	0.000531	80.1	0.0005289	-0.5
16	[9], Tabl.8, № 1	150	3	0.5	6	1.5	0.0003645	108.54	0.0003136	-14.0
17	[9], Tabl.9, № 4	50	3	0.06	50	0.18	0.0002793	104.4	0.0003351	+20.0
18	[9], Tabl.9, № 3	100	3	0.2	15	0.6	0.0002841	114.4	0.0002863	+0.7
19	[15], Tabl.2, №3	100.3	3	0.2	15	0.6	0.003088	114.7	0.0002848	-7.77
20	[9], Tabl.8, № 2	150	3	0.37	8.11	1.11	0.0002403	126.18	0.0002416	+0.6
21	[9], Tabl.8, № 5	120	3	0.2	15	0.6	0.000218	137.3	0.0002086	-4.3
22	[9], Tabl.9, № 2	120	3	0.2	15	0.6	0.0002045	137.3	0.0002086	-2.0
23	[9], Tabl.9, № 2	120	3	0.2	15	0.6	0.0002045	137.3	0.0002086	-2.0
24	[9], Tabl.8, № 3	150	3	0.2	15	0.6	0.0001432	171.6	0.0001412	-1.4
25	[9], Tabl.9, № 1	150	3	0.2	15	0.6	0.0001432	171.6	0.0001412	-1.4
26	[15], Tabl.2, № 1	150	3	0.2	15	0.6	0.0001353	171.6	0.0001412	+4.4
27	[9], Tabl.8, № 4	150	3	0.1	30	0.3	0.000074	242.7	0.0000767	+3.6

Fig.2 presents the results of computing according to formulae (4) and (7'), (7'') the dependencies $N(\lambda')$ for rectilinear 3-, 4- and 6-gons and plates in the range of λ' variation $0 \le \lambda' \le 10$.

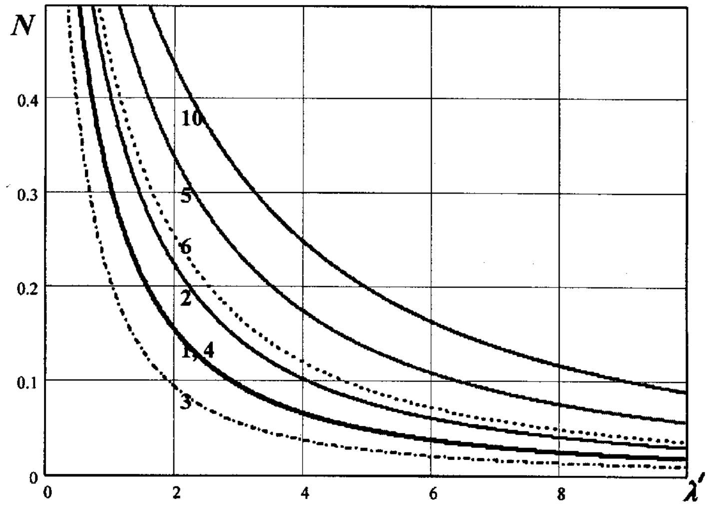

Figure 2. Results for calculating the dependencies $N(\lambda')$ according to formulae (4) and (7'), (7") for rectilinear 3-, 4- и 6-gons (respectively the curves 3, 4, 6) and plates at ratios of the sides of their cross-section γ =1, 2, 5, 10 (respectively curves 1, 2, 5, 10).

Supplement 4: Hollow cylinder, magnetized longitudinally. Let us characterize the dimensions of a hollow cylinder by parameters $\lambda = L/D$ and $h = 2H/D$, where L is length, D is the outer diameter, and H is the width of the wall.

Further we analyze the known methods and the corresponding formulae for computing N of a hollow cylinder, using the proposed notations.

In [7, 16, 13], the computation techniques are recommended for N of hollow bars with the formulae for the solid cylinder, the sectional area of which equals the area of metal in the cross-section of the bar. These techniques allow to obtain the following formula for computing N of a hollow cylinder:

$$N \approx Э(\lambda_1 эф) \cdot k(\lambda_1 эф), \qquad \text{where} \quad \lambda_1 эф = \frac{\lambda}{\sqrt{h \cdot (2-h)}} \quad . \tag{8}$$

In [8], the authors introduced an approach for computing the permeability of a hollow cylinder using the formulae for a solid cylinder of an equivalent perimeter. Combined with the formula (4) for calculating N for solid cylinders, this approach allowed to obtain [1] the following formula for computing N of hollow cylinders:

$$N \approx h \cdot (2-h) \cdot Э(\lambda) \cdot k(\lambda) \quad . \tag{9}$$

In [17], the calculation of demagnetizing field on the axis of a hollow cylinder is accomplished using a “conventional” demagnetization coefficient N'. This coefficient equals the difference of N of solid cylinders, which have the diameters equaling the outer and the inner diameters of the hollow cylinder. This approach allowed to obtain for N' of hollow cylinders:

$$N' = Э(\lambda) \cdot k(\lambda) - Э(\overline{\lambda}) \cdot k(\overline{\lambda}) \quad , \qquad \text{где} \quad \overline{\lambda} = \frac{\lambda}{1-h} . \tag{10}$$

In [18, 19], the hollow cylinder for computing N is replaced by an equivalent hollow ellipsoid. The formula obtained in [18, 19] for computing N of hollow cylinders made of materials with high magnetic permeability $(\mu \to \infty)$ can be written as:

$$N \approx 0.765 \cdot h \cdot \sqrt[3]{(2-h)\cdot(3-3h+h^2)^2} \cdot Э(\lambda) \quad . \tag{11}$$

In the Table 4 the results of computing N of hollow cylinders according to (8) - (11) are compared to the known [9, 11] experimental results at ($0.6 \le \lambda \le 50$ and $0.1 \le h \le 0.5$). The error of computing N of hollow cylinders according to (8) - (11) is calculated with (6).

Table 4

Comparison of results of measuring N of hollow cylinders for [9] and [11] with calculation according to (8) - (11).

№ пп	Data source	Dimensions of samples, mm			Generic parameters		N Experiment	Deviation σ, % of the computation results from experiment			
		L	D	H	λ	h		Computing acc (8)	Computing acc (10)	Computing acc (11)	Computing acc (9)
1	[11]	25.5	41.7	5.1	0.612	0.245	0.2037	+44.3	-61.7	-8.4	-14,0
2	[11]	20.1	20.2	5.1	0.995	0.505	0.2208	+10.5	-48.4	-3.1	-4.6
3	[11]	40.7	30.0	2.75	1.357	0.183	0.0634	+72.6	-37.8	+26.6	+5.8
4	[11]	39.0	26.2	3.1	1.489	0.237	0.0755	+51.0	-36.4	+20.2	+3.3
5	[11]	59.0	26.2	3.1	2.252	0.237	0.0457	+46.6	-27.8	+28.8	+7.5
6	[11]	85.9	26.2	3.1	3.416	0.237	0.0270	+40.7	-22.2	+32.4	+7.4
7	[9]	76.0	2.7	0.15	28.15	0.111	0.000641	+29.2	-18.4	+21.1	-1.6
8	[9]	60.0	2.0	0.10	30.0	0.100	0.000551	+24.1	-22.5	+15.1	-6.5
9	[9]	100	2.0	0.10	50.0	0.100	0.000232	+22.8	-22.8	+14.6	-4.2

Analysis of the obtained results shows, that a decent agreement with experimental results in the studied variation ranges of λ and h is achieved by computing N of hollow cylinders with (9). The deviation of the computed results from the experiment does not exceed ± 7.5 % . This value is in the scope of the possible experimental error.

Comparison of the results of computing N of hollow cylinders according to (9) with experimental results, introduced in [20, 21] is provided in the Table 5.

Table 5

Comparison of results of computing N of hollow cylinders according to (9) with experimental results of [20]* and [21]**

h / λ	1	0.8	0.6	0.4	0.3	0.2	0.1
0.2	**0.709**	**0.681**	**0.595** 0.5**	**0.452** 0.4**	**0.362**	**0.255** 0.26**	**0.135**
0.4	**0.532**	**0.51**	**0.446** 0.36**	**0.34** 0.29**	**0.271**	**0.191** 0.17**	**0.101**
1	**0.276** 0.257*	**0.265** 0.2509*	**0.232** 0.2161* 0.19**	**0.177** 0.1726* 0.15**	**0.147** 0.1296*	**0.099** 0.0975* 0.082**	**0.053** 0.0483*
2	**0.134** 0.125*	**0.129** 0.1213*	**0.113** 0.1053* 0.1**	**0.086** 0.0814* 0.08**	**0.069** 0.0633*	**0.048** 0.0451* 0.042**	**0.026** 0.02416*
3	**0.082** 0.0782*	0.079 0.0741*	**0.069** 0.0649*	**0.052** 0.05*	**0.042** 0.0391*	**0.029** 0.02807*	**0.016** 0.01477*
4	**0.056**	**0.054**	**0.047** 0.05**	**0.036** 0.038**	**0.029**	**0.020** 0.021**	**0.011**
5	**0.041** 0.0407*	**0.040** 0.0389*	**0.035** 0.0337*	**0.026** 0.02621*	**0.021** 0.02009*	**0.015** 0.01459*	**0.0078** 0.00797*
7	**0.026** 0.0254*	**0.025** 0.0245*	**0.021** 0.02133*	**0.016** 0.01592*	**0.013** 0.01235*	**0.0092** 0.00995*	**0.00485** 0.00462*
10	**0.015** 0.01495*	**0.014** 0.01455*	**0.013** 0.01249*	**0.0097** 0.00943*	**0.0077** 0.00721*	**0.0054** 0.00527*	**0.00287** 0.002372*
15	**0.0814** 0.0794*	**0.007816**	**0.006839**	**0.0052** 0.005*	**0.004152** 0.00375*	**0.00293** 0.002499*	**0.001547**

These results show that formula (9) with a sufficient for practical use precision describes all experimental data points, including those at $\lambda = 0.2$. The latter is important, because at $\lambda \to 0$, the computed by (9) value of N does not approach 1 for any $h \neq 0$, but the value $h \cdot (2 - h)$, which is not correct physically. Nevertheless, this fact does not affect the high accuracy of the account for the experimental data (Tables 4 and 5) for measuring N of hollow cylinders in the practically relevant variation range of their λ ($0.2 \leq \lambda \leq 50$).

On the Fig.3, the results of computing according to (9) the N of hollow cylinders with varying width of the wall at $0.2 \leq \lambda \leq 10$ are compared with the results of computing N of rotation ellipsoids and solid cylinders.

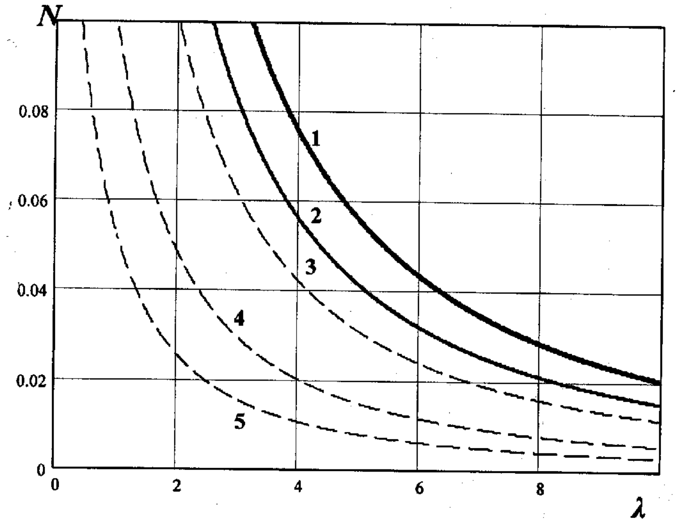

Fig 3. Result of computing N for rotation ellipsoid (1), solid cylinder (2) and hollow cylinder at h = 0.5, 0.2 and 0.1 (correspondingly curves 3, 4 and 5). Computing according to (1), (4) and (9).

Supplement 5: Hollow cylinder, magnetized orthogonally to the axis. Analysis of the formulae for calculating the N_∞ of hollow cylinders from materials with high magnetic permeability (ʃ >> 1) magnetized orthogonally to the axis is provided in [2]. It is shown, that for calculating the N_∞ in the whole possible variation range of their λ and h ($0 \le \lambda < \infty$, $0 \le h \le 1$), the following formula can be recommended:

$$N_\perp \approx \frac{h}{2}\left[1 - Э(\lambda)k(\lambda)\right] \qquad \text{для } 0 \le \lambda < \infty,\ \ 0 \le h \le 1 \ . \qquad (12)$$

Supplement 6: Interpolating formula for calculation the ***N* of short hollow cylinders magnetized longitudinally** is elaborated in [3]**.** It is shown, that for computing N for hollow cylinders from materials with high magnetic permeability (ʃ >> 1) magnetized longitudinally in the variation ranges of their λ and h ($0 \le \lambda \le 0.2$, $0 \le h \le 1$), the following formula can be recommended:

$$N \approx \frac{1}{1 + \beta \cdot \lambda^{\varepsilon}}\ , \qquad (12)$$

где $$\varepsilon = 0.3075 \cdot \left[1 + \frac{1}{\frac{1.41}{h\cdot(2-h)} - 1}\right], \qquad \beta = 5^{\varepsilon} \cdot \left[\frac{1.41}{h\cdot(2-h)} - 1\right]. \qquad (13)$$

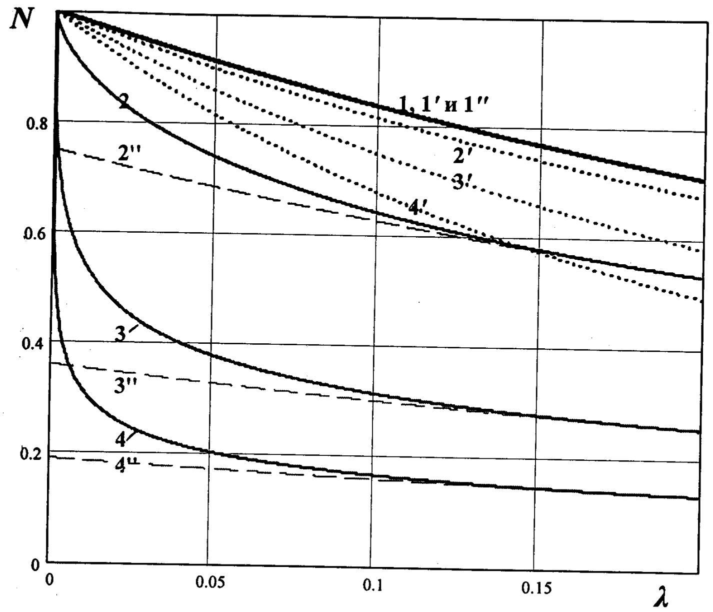

Figure 4. Results of computing with the formulae: (8) - the dashed curves, (9) – double-dashed curves, and (12) – solid curves, -- of the dependencies $N(\lambda)$ at $0 \leq \lambda \leq 0.2$ for $h = 1$ (curves 1, 1′ and 1″ coincide), $h = 0.5$ (2, 2′ and 2″), $h = 0.2$ (3, 3′ and 3″), and $h = 0.1$ (4, 4′ and 4″).

References

1. Sandomirski S.G. Recommendations on use in technical calculations of formulas for central demagnetization coefficient of solid and hollow cylinders, bars and plates made of materials with high magnetic permeability (review). – Technical diagnostics and non-destructive testing (Техническая диагностика и неразрушающий контроль), 2008, № 3, pp. 38 – 46.

2. Sandomirski S.G. Analysis of formulas for calculating the central demagnetization coefficient of hollow cylinders made of a material with high magnetic permeability at magnetization orthogonally to the generatrix. . – Electrotechnics (Электротехника), 2008, № 3, pp. 45 – 51.

3. Sandomirski S.G. Computing short hollow cylinders at magnetization parallel to the generatrix. – Electrotechnics (Электротехника), 2009, № 2, pp. 52 – 55.

4. Arkadiew W. Magnetisierungskoefficienten der Gestalt, des Stoffes und des Korpers. - The Journal of Russian Physico-Chemical Society (Журнал Русского физико-химического общества). V. 46, Physics section, issue 1, 1914, pp. 22 – 42.

5. Matuk V.F., Osipov A.A. Some remark about demagnetizing factor of different shape articles. I. Coefficient of demagnetizing of ellipsoids and cylinders. – Defectoscopy (Дефектоскопия), 1999, № 7, pp. 41 – 49.

6. Wurschmidt J. Die Entmagnetisierungsfaktoren kreiszylindrischer Stäbe. Zeitschr. f. Physik, v. 19, (1923), pp. 388 - 397.

7. Schneider W. Untursuchungen über Magnetisierungskurven und Vergrößerung der Empfindlichkeit des Scheringschen Deflektorenmagnetometers. Zeitschr. f. Physik, v. 42, H.11/12 (1927), pp. 883 - 898.

8. Vitol V.G. On demagnetizing factor and the non-decisive similarity coefficients for magnetizing cylinders. – Transactions of the Institue of Physics AS of Latland SSR, v.YII, 1954, pp. 45 – 68.

9. Rosenblat M.A. Demagnetization coefficients for bars with high permeability. - Journal of technical physics (ЖТФ), 1954, v. XXIV, issue 4, - pp. 637-661.

10. Warmuth K. Über den ballistichen Entmagnetisierungsfaktor zylindrischen Stäbe. – Archiv fur Elektrotechnik, 1954, т.41, Nr. 5, pp. 242 – 257.

11. Slivinskaya A.G. Permeability of shapes of cylinders and prisms. – Transactions of the Moskau Energy Institute, issue 16, 1956, pp. 67 – 81.

12. Burtsev G.A. Calculation of the Demagnetized Coefficient of Cylindrical Rods. – Defectoscopy (Дефектоскопия), 1971, № 5, pp. 20 - 30.

13. Meskin V.S. Ferromagnetic alloys. M.-L.: ОНТИ, 27, 1937 (In Russian).

14. Dietz G., Meingast R. Ein ferromagnetischer Stab in homogenen Minetfeld. – Zs. F. Ang. Physik, 1971, v.31, N 1, S.77-82.

15. Rosenblat M.A. Ballistic demagnetization coefficient of bars with rectangular profile.- JTP (Журнал технической физики), v.XX, 1950, issue 9, pp. 1117 – 1120.

16. Arkadiew W. Electromagnetic processes in metal's. - M.-L., ОНТИ, 1934, part 1 (In Russian).

17. Gorbash V.G., Sandomirskii S.G., Delendik M.N. Factor of demagnetization of hollow ferromagnetic rods. – Technical diagnostics and non-destructive testing (Техническая диагностика и неразрушающий контроль), 1999, № 2, pp. 9 – 15.

18. Misiuk L.Y., Nichoga V.A. On calculation of demagnetization coefficients for hollow cyllindrical cores. – Geophysical equipment (Геофизическая аппаратура), issue 25, 1965, L., «Недра», pp. 70 – 98.

19. Misiuk L.Y., Nichoga V.A. Analytical Relationships for Calculating the Demagnetizing Coefficients of Cores. – Electricity (Электричество), 1967 г., № 10, pp. 73 – 74.

20. Matuk V.F., Osipov A.A., Strelukhin A.V. Central demagnetizing factor of hollow cylindrical rods made of soft magnetic materials. – Defectoscopy (Дефектоскопия), 2007, № 3, pp.26 - 36

21. Okoshi T. Demagnetizing Factors of Rods and Tubes Computed from Analog Measurements. – J. Appl. Physics, 1965, v.36, N 8, pp. 2382 – 2387.